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Recession Characteristics of Groundwater Outflow and Base Flow From Mountainous Watersheds

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The relationship between base flow recession characteristics in steep watersheds and geomorphologic and soil parameters is investigated. The formulation for the groundwater outflow was obtained by means of a hydraulic approach applied to a simple conceptual model for a hillslope. Long-term flow data of 19 representative basins in the Allegheny Mountain section of the Appalachian Plateaus were analyzed on the basis of this formulation. Results showed that the reaction factor, which is a time scale of base flow recession, is dependent on the mean land slope, the drainage density, and the ratio (K/f) of the hydraulic conductivity and the drainable porosity. On account mainly of the nonuniform distribution of the physical characteristics within a basin, the reaction factor for a given watershed is somewhat variable with time, but the adoption of a constant value is useful to represent average conditions for a recession period. Analysis of the (K/f) dependency showed that macropores and other structural features may greatly affect the watershed base flow. Evaporation from groundwater appears to constitute only a minor portion of overall basin evaporation.

INTRODUCTION

During rainless periods, the flow in a basin's stream system, which is often referred to as base flow is sustained by groundwater discharge. But even during wet periods, base flow constitutes an important and sometimes dominant streamflow component particularly in humid forested areas. Therefore an understanding of the behavior of groundwater outflow is essential in studies of water budgets and the response of catchments to various hydrologic and climatic inputs.

The natural groundwater discharge in a catchment as a function of time is largely controlled by the physical and hydrologic properties of the aquifer materials, many of which are reflected in the morphology of the basin. Since geomorphologic characteristics can be readily obtained from maps and air photos, it is of interest to establish reliable relationships between the groundwater outflow rate and the controlling geomorphologic parameters of a basin. As noted elsewhere [*Zecharias and Brutsaert*, this issue], previous hydrogeomorphic studies have been primarily statistical in nature. On the other hand, in groundwater hydrology, most investigations that deal with the problem of drainage from hillslopes have focused on the hydraulics and almost no attempts have been made to relate the outflow to basin-wide geomorphologic or aquifer parameters. The objective of this paper is to investigate the effect of geomorphologic and soil characteristics on base flow recession by means of a simple conceptual model and on the basis of basin scale parameters. The parameters in question are mainly land slope and drainage density of watersheds, but the hydraulic conductivity and the drainable porosity of the aquifer materials are also considered. The approach is then applied to data obtained for some basins located in the Appalachian Plateaus.

DESCRIPTION OF STUDY AREA

This study makes use of hydrologic and geomorphologic data of some representative watersheds in the Allegheny Mountain section of the Appalachian Plateau physiographic province. The section consists of a sequence of near-horizontal paleozoic sedimentary rocks mainly conglomerate, sandstone, and shale [*Thornbury*, 1965, p. 130)]. The rock formations are only mildy deformed and form broad, open anticlinal and synclinal structures with a gentle regional dip; consequently, the evolution of drainage systems in the area has not been strongly affected by structural controls. Throughout the section, which is unglaciated, fluvial erosion has created a deeply and irregularly dissected topography that is characterized by steep slopes and V-shaped valleys. Average elevations of mountain summits range from 700 to 830 m and, depending on their size, stream valleys may be up to 300–600 m deep [e.g., *Morisawa*, 1959]. As a result, the section constitutes one of the highest and most rugged parts of the Appalachian Plateau province [*Atwood*, 1940, p. 115; *Hunt*, 1967, p. 175]. The climate is humid continental with an average annual precipitation of about 1000–1500 mm. Most of the precipitation is delivered between early spring and late summer and supports a dense vegetative cover 75% of which is forest and 10% cropland [*Baker and Dill*, 1971].

Nineteen watersheds, within the Allegheny Mountain section were selected as study basins. To avoid the need of using channel routing techniques, basin area was restricted to about 200 km^2. All the basins were at one time or another gaged by the U.S. Geological Survey and have therefore long-term flow records. These records indicate that the streamflow data are classified as good and that the flows were not subjected to regulation or diversion. The basins are well-distributed within the Allegheny Mountain section and each contains and/or is located in the vicinity of a number of meteorological stations whose records are contemporaneous with the streamflow data.

A notable topographic feature of the basins is the absence of flood plains in their upland regions. First- and second-order streams occupy the bottom of V-shaped valleys and a significant fraction of the narrow flood plains associated with third- and fourth-order streams is occupied by the stream channels themselves. Soil Survey reports of the Counties in which the basins are located show that the bedrock is mantled by a weathered layer which has a nearly uniform thickness of about 1.5–2.0 m. The remarkable uniformity of soil thickness in the

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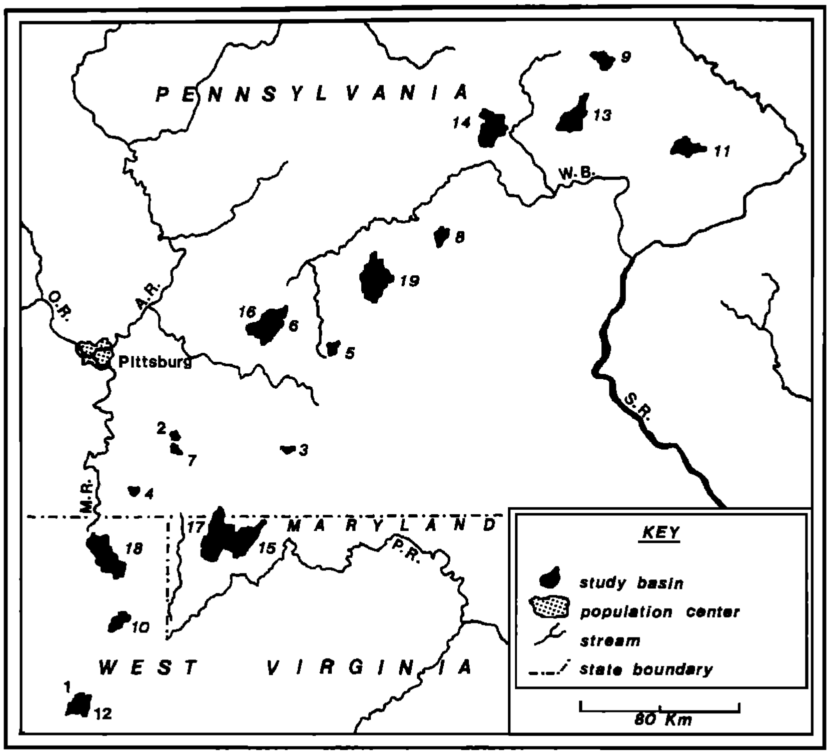

Fig. 1. Study basins and associated drainage systems: A. R., Allegheny River; M. R., Monongahela River; O. R., Ohio River; P. R., Potomac River; W. B., West Branch Susquehanna River.

study watersheds as well as this average soil thickness can be observed along stream banks and road cuts at various topographic levels. In all the watersheds, this layer forms an unconfined aquifer. The size, location, and geographic distribution of the study basins are shown in Figure 1 (see also *Zecharias and Brutsaert* [1985]).

DATA

Streamflow Data

The long-term streamflow data of the basins, from which their base flow hydrographs were derived, were obtained from the appropriate Water Supply Papers published by the U.S. Geological Survey. Daily temperature records of meteorological stations that are located in and/or near the study basins and other related information [see *Baker and Dill*, 1971] indicate that frozen ground conditions may exist in the area during the period October to May inclusive. The shallowness of the unconfined aquifers in the area and the effect that freezing may have on the natural groundwater flow regime made it necessary to exclude the parts of the streamflow records that correspond to these months of the year. On the basis of rainfall records of weather stations associated with each study basin (see Meteorological Data below), all daily streamflows on rainy days were next excluded to obtain the dry period or low flow data for the basin. The data thus screened may still contain a "direct" runoff component from precipitation that occurred in preceding days. Therefore in addition, the flow for the first day of every dry period was excluded and the resulting data were assumed to constitute a base flow record for the basin.

Meteorological Data

Through the conjunctive use of the location maps of the study basins and those of meteorological stations in the area, a number of (not fewer than four) weather stations located in or around each basin was identified. Daily average values of temperature, rainfall, and snowfall of these stations for periods covered by the flow records of the associated basins were then obtained from published records of the National Oceanic and Atmospheric Administration.

Geomorphologic Data

The geomorphologic parameters that affect groundwater outflow were generated from recent 1:24,000 U.S. Geological Survey topographic maps. The methods or formulas of computing them have been described elsewhere [see *Zecharias and Brutsaert*, 1985, this issue]. The drainage density values are presented in Table 1 and the land slope values are given by *Zecharias and Brutsaert* [1985].

Soil Data

The U.S. Department of Agriculture County Soil Survey reports provide data on depth to bedrock (i.e., soil thickness), soil hydraulic conductivity, and ground surface slope all of which are given as average values for each soil unit in an area. In addition, the reports include information on the texture and stratification of soil units as well as maps showing all mappable units. For the present study, the outline of each basin was transferred from topographic maps on to corresponding soil maps, and the fraction of basin area that each unit comprises was measured. These values and horizon thicknesses of soil units were then used as weighting factors to obtain basin averages of hydraulic conductivity and depth to bedrock. Furthermore, it was possible to make a qualitative assessment of the dominant grain size and texture of the soils in each study basin. This information together with the "grain size—specific yield graphs" developed by *Johnson* [1967] allowed the estimation of the average drainable porosity for

TABLE 1. Drainage Density and Average Parameters of Soils in the Study Basins

			Soil Parameters		
No.	Drainage Basin	Drainage Density, km^{-1}	Hydraulic Conductivity K, cm/hour	Soil Depth D, m	Drainable Porosity f
1	Grassy Run	1.6871	3.37	1.04	0.20
2	Green Lick Run	1.2727	5.50	1.70	0.15
3	Clear Run	1.2152		1.52	
4	Lick Run	1.2574	13.50	1.10	0.25
5	Bradley Run	1.0430		1.45	
6	Little Yellow Creek	1.3879	7.96	1.38	0.20
7	Poplar Run	1.1172	5.22	1.62	0.25
8	South Fork Beech Creek	0.8450	9.11	1.65	0.15
9	Corey Creek	0.9492	4.80	1.22	0.15
10	Buffalo Creek	1.2038	5.00	1.31	0.15
11	Muncy Creek	1.0018		1.25	
12	Roaring Creek	1.6081	4.50	1.52	0.20
13	Blockhouse Creek	1.0857	7.20	1.45	0.20
14	Young Womans Creek	0.9007	4.10	1.40	0.15
15	Savage River	1.1510	6.90	1.50	0.20
16	Yellow Creek	1.2702	7.41	1.46	0.20
17	Casselman River	0.7517	5.60	1.45	0.15
18	Deckers Creek	1.2591	5.20	1.39	0.15
19	Moshannon Creek	0.9442	3.90	1.50	0.15

each basin. The resulting basin average values are presented Table 1.

ANALYSIS

Formulation of Outflow Rate

The geologic and geomorphologic setting of the study basins, their soil characteristics and reconnaissance field observations were used as a basis to establish a simple conceptual model. The cross section shown in Figure 2 represents a shallow, unconfined aquifer which has a small depth-to-length ratio $D \cos \theta / B$ which rests on an inclined impermeable layer (IL) representing the underlying bedrock. The two ends of the aquifer model correspond to the groundwater divide and the bank of a fully penetrating stream.

Drainage from an inclined slab of porous material involving both saturated and unsaturated flow can be described by Richards's equation [e.g., *Brutsaert and El-Kadi*, 1984, 1986]. But this equation can only be solved numerically, and as a result, the approach cannot easily be implemented to derive a parametric equation suitable for the present purpose. More appropriate is the hydraulic approach based on the following equation

$$q = -Kh[(dh/dx) \cos \theta + \sin \theta] \qquad (1)$$

where $q[L^2/T]$ is the flow rate per unit width of aquifer, K the hydraulic conductivity, h the thickness of the saturated zone measured perpendicular to the IL, and θ the slope angle of the IL.

Many studies devoted to the problem of hillslope drainage have made use of the hydraulic approach by means of (1) which was first developed by *Boussinesq* [1877; *Childs*, 1971]. However, in most of these studies [e.g., *Henderson and Wooding*, 1964; *Beven*, 1981] the approach was further simplified by the kinematic wave approximation. Also, *Sloan and Moore* [1984] and *Stagnitti et al.* [1986] used this approximation to describe hillslope drainage as partly saturated flow. Because the hydraulic gradient is assumed to be sin θ and the (dh/dx) term in (1) is neglected, for any horizontal aquifer this approximation produces a zero flow. As a result, the kinematic wave approach is unsuitable when a wide range of aquifer slopes, including very small ones, has to be considered.

In the present study, (1) and the physical model shown in Figure 2 were used to derive a simple outflow equation by means of the quasi steady state approach. This approach was pioneered around 1886 by K. E. Lembke (cited by *Polubarinova-Kochina* [1962, p. 573]) for an infinitely long aquifer and it has been applied successfully by *DeZeeuw* [1979; also *Kraijenhoff*, 1979, p. 315] for a horizontal aquifer of finite length. Interestingly, this same approach was applied by *Landahl* [1953] in the solution of linear diffusion and later by *Macey* [1959] to a more general class of nonlinear diffusion problems. *Parlange* [1971] used it in the study of horizontal infiltration. The accuracy of the quasi steady state approach for sorption problems has been examined by *Brutsaert* [1976].

As shown in the appendix, the application of the approach to sloping aquifers results in the equation

$$q = (q_0 + EB)e^{-\alpha t} - EB \qquad (2)$$

where q is the outflow rate per length of stream channel at $x = 0$, q_0 its initial value, E the mean (i.e., constant) evapora-

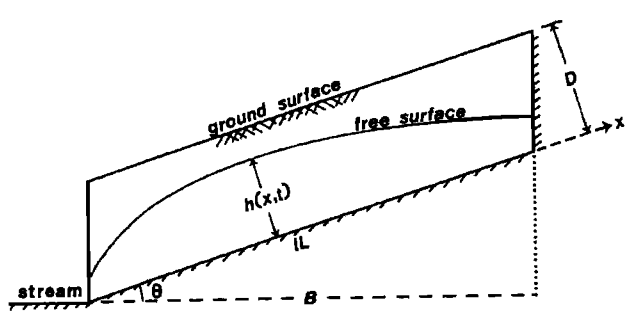

Fig. 2. An idealized physical model of a hillslope. B, horizontal aquifer length; D, aquifer thickness; IL, impermeable layer; θ, inclination angle of impermeable layer.

tion rate, and B the horizontal projection of the aquifer length. The parameter α is the aquifer reaction factor and is given by

$$\alpha = 2K(pD \cos^2 \theta + B \sin \theta)/(fB^2) \quad (3)$$

in which f is the drainable porosity, D the mean aquifer depth, and p is a constant arising in the linearization.

It should be noted that the aquifer reaction factor is a convenient measure of the groundwater outflow. This results from the fact that the total groundwater outflow volume (hence total aquifer storage) is the integral of the outflow rate namely, $\int_0^\infty q\, dt = q_0/\alpha$, where q_0, the initial outflow rate, is dependent only on precipitation input. Therefore for a given input, groundwater discharge volume is solely a function of the aquifer reaction factor α. Put differently, α^{-1} is a characteristic time scale for duration of groundwater outflow.

Hydrologic Application

The base flow recession characteristics of the basins can be determined by means of a method proposed by *Brutsaert and Nieber* [1977]. The method is based on the general form in which most groundwater outflow equations can be expressed, namely,

$$dQ/dt = f(Q) \quad (4)$$

where Q is the outflow rate from the basin at the gaging station. The functional relationship $f(Q)$ may be determined from a plot of $|dQ/dt|$ versus Q. For actual flow data this consists of plotting $|(Q_i - Q_{i-1})/\Delta t|$ against $(Q_i + Q_{i-1})/2$, where Q_i is the flow rate at time t and Q_{i-1} is that at $t - \Delta t$. One advantage of this method is that it eliminates the problem of determining the time reference $t = 0$ after each interruption of base flow recession by precipitation. Moreover, as one is only concerned with the rate of change of discharge from one day to the next, the length of a given recession period is not of primary importance. If α and E can be taken as average or characteristic basin scale values, q can be readily related to Q by integrating it along all base flow contributing stream channels in the basin. The characteristic basin scale value of B can be taken as [see *Horton*, 1932; *Brutsaert and Nieber*, 1977] one half the ratio of the watershed area to the total length of streams (A/L), i.e., one half the inverse of the drainage density. Then (2) can be put in the form of (4) as follows:

$$dQ/dt = -\alpha(Q + EA) \quad (5)$$

where A is the watershed area. With Δt as 1 day, average daily flows of recession periods were plotted for each study basin. According to (5) the aquifer reaction factor for the watershed is the slope of the regression line, and the evaporation rate can be computed by using both the slope and the intercept of the line. The values of α (denoted by α_r) and of E, determined by applying the method of least squares to (5), are given in Table 2.

The values of $E\,A$, obtained this way, are very small, and probably on the order of the Q data, or even smaller. Therefore values of α were also determined by assuming $E = 0$ in (5), that is, by forcing the best fit straight line for each basin through the origin. The fit was carried out in two ways. The first was determined by the method of least squares; the resulting values of the reaction factor are shown in Table 2 as α_{r0}. The second method was graphical. For each basin the reaction factor was taken as the average of the slopes of the straight line upper and lower envelopes of the dQ/dt versus Q data points through the origin; to exclude possible anomalous outliers, up to 10% of the points were allowed to fall outside the upper and lower envelopes. These values are denoted as α_e in Table 2.

DISCUSSION OF RESULTS

The results given in Table 2 are obtained by means of standard statistical and graphical analyses of (5). However, the form of (5) is derived on the basis of strictly physical considerations. These results can provide insight into several issues.

Linearity of Lumped System

The values of α (i.e., α_r, α_{r0}, and α_e) and E given in Table 2 are basin scale, time average values obtained from the application of the exponential decay base flow equation (5). As an illustration, the data points for Savage River are shown in Figure 3. The plot displays considerable scatter indicating that for a given value of Q, the value of α is not a unique constant, but probably a variable depending on hydrologic conditions in the basin. Undoubtedly, part of this scatter is due to noise in the flow data, but part of it must be due to limitations in the analysis which was based on conceptualizing the watershed as a single lumped and linear storage unit.

To analyze the variation of α with time, several plots of the base flow data of a given basin were prepared each time by using only those flows that occurred after a successively larger number of days following the beginning of every recession period. In other words, the $|dQ/dt|$ versus Q data, corresponding to a given state of the recession process, were plotted after they were isolated from those of the preceding days. This way it became possible to examine how the recession behavior changed 2, 4, 6, and 7 days into the dry period. This graphical analysis was performed by means of the upper and lower envelopes of the points. Clearly, the upper and lower envelopes represent the maximal and minimal observed rates, respectively, of the recession of the groundwater outflow. Note that this graphical analysis was carried out as indicated earlier for α_e; thus for convenience the envelopes were taken through the origin, and, to allow for possible error in the data, outliers were excluded by letting up to 10% of the points fall outside the upper and lower envelopes.

To illustrate the procedure, the successive plots for Savage River have been condensed into a single composite plot in Figure 3. For the plots of flow values that occurred after 2, 4, 6, and 7 days following a rainfall event, the slopes of the upper envelopes are 0.33, 0.23, 0.19, and 0.15 and those of the lower envelopes 0.062, 0.062, 0.063, and 0.066, respectively. Thus the slopes, i.e., α, of the lower envelopes remain approximately constant, while those for the upper envelopes decrease continuously with time. The value of Q depends mainly on the storage of water in the watershed. But the upper envelope provides information on the groundwater outflow regime in the early stages of a dry period when the rates of recession, i.e., $|dQ/dt|$ are high. The successive values of α show that there are aquifers in the basin whose reaction factors are initially large, but decrease sensibly as the rainless period continues. Advanced states of the outflow process, which are accompanied by small recession rates, are represented by the lower envelope whose slope in the successive scatter diagrams remains essentially the same. This suggests the presence of aquifers whose α

TABLE 2. Reaction Factor α and Evapotranspiration E Values Determined for the Study Basins

No.	Drainage Basin	α_r (day^{-1}) From Flow Data Regression With (5)	Evaporation E (mm/day) From Flow Data Regression With (5)	α_{r0} From Flow Data Regression With (5) for E=0	α_c From Flow Data Average Envelopes Slopes	Predicted $\alpha_p \times 10^2$ by Means of (3)
1	Grassy Run	0.1159	0.1602	0.1054	0.1145	0.466
2	Green Lick Run	0.2388	0.0326	0.2304	0.2132	0.860
3	Clear Run	0.1693	0.0961	0.1512	0.1207	
4	Lick Run	0.2995	0.0896	0.2717	0.2452	2.355
5	Bradley Run	0.1608	0.3099	0.1206	0.1117	
6	Little Yellow Creek	0.3061	0.1387	0.2602	0.1961	0.858
7	Poplar Run	0.3022	0.0655	0.2723	0.2121	0.427
8	South Fork Beech Creek	0.1762	0.2592	0.1299	0.1265	0.679
9	Corey Creek	0.1658	0.0153	0.1584	0.1581	0.721
10	Buffalo Creek	0.3388	0.0614	0.3096	0.2168	1.292
11	Muncy Creek	0.2200	0.1802	0.1789	0.1562	
12	Roaring Creek	0.2799	0.0467	0.2593	0.2487	1.812
13	Blockhouse Creek	0.1559	0.0517	0.1392	0.1327	0.995
14	Young Womans Creek	0.1343	0.0876	0.1195	0.1123	0.471
15	Savage River	0.2498	0.0631	0.2238	0.1959	1.060
16	Yellow Creek	0.1324	0.0068	0.1308	0.1043	0.995
17	Casselman River	0.1777	0.0479	0.1649	0.1379	0.421
18	Deckers Creek	0.2939	0.0464	0.2700	0.2542	1.319
19	Moshannon Creek	0.1347	0.2455	0.1032	0.1023	0.612

although relatively small, remain nearly constant throughout most of the recession period.

It is likely that the variation of α with time is largely the result of the nonuniform distribution of the physical characteristics within a watershed. Equation (3) shows that the steeper parts of a basin, where θ is large and where B tends to be smaller (and perhaps K/f larger), must have fast depletion rates; such areas are generally located in the headwater sections of a basin. In contrast, the downstream regions of a basin have smaller inclinations, hence relatively lower rates of depletion. As indicated earlier, the layer of weathered material in the study area is fairly uniform; therefore D does not seem to be a factor causing substantial α variability.

These results show that while the characterization of a basin as a single lumped unit with basin scale parameters is a useful approximation, it can have certain limitations. The total groundwater discharge is the sum of flow contributions of aquifer sections which have unequal reaction factors. This total flow is initially dominated by the discharges of the steeper parts which contribute a large fraction of the total flow during the first few days of a recession period. As the recession progresses, however, their storage decreases rapidly and the gentler parts of the aquifer, now being the major contributors, determine the outflow.

Effect of Basin Scale Characteristics

The lumped approach (see the appendix) indicated that the reaction factor α is a function of the basin scale parameters that appear on the right-hand side of (3). Three different sets of α values of the study basins (i.e., α_r, α_{r0}, and α_c) have already been determined from the flow data by means of (5), and the other parameters in (3) are known quantities. Therefore to assess the relative strengths of the effects of the parameters, their respective partial correlations with these three sets of α values were computed. Because the highest correlations were obtained with α_r, only these results are given here (see Table 3). However, the other correlations were similar. To test the sensitivity of these results, partial correlations were also computed by using slightly different sets of variables, e.g., θ, fB^2, and K. In addition, the tests were repeated after successive elimination of larger watersheds. (It was suspected that the conceptual model might perhaps be more applicable for

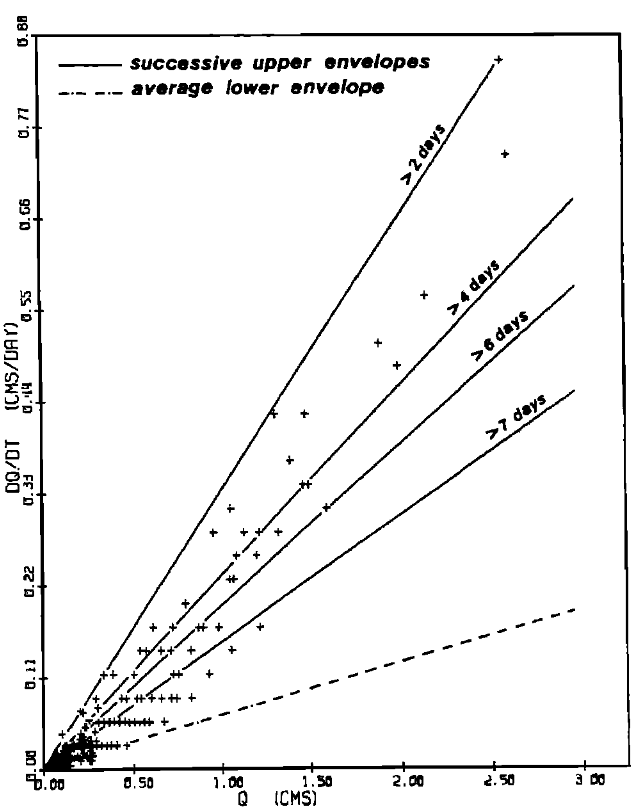

Fig. 3. A plot of the base flow data showing envelopes that correspond to successive stages of recession periods (Savage River basin, Maryland).

TABLE 3. Partial Correlations Between Variables Shown and the Reactions Factor α_e

Variable	Partial Correlation Coefficient	Confidence Level Significant at
K/f	0.32	73%
B	−0.37	92%
$T=D \cos^2 \theta/B \sin \theta$	0.52	98%

smaller basins). However, the results were not significantly different from those given and are therefore not presented. The results shown in Table 3 indicate that the basin ground surface slope term T exerts the strongest effect on the magnitude of α. A little smaller, but practically just as significant, is the effect of the basin aquifer width B (or stream density). The effect of the soil parameter K/f is smallest and not very significant.

The basin scale morphologic/soil characteristics can also be used to predict α by means of (3). The α values predicted this way (with $p = 1$) are given in Table 2 as α_p. To assess the reliability of (3) as a predictor of α, the predicted α_p values and the values obtained directly from the flow data (i.e., α_r, α_{ro}, α_e) were subjected to correlation tests after the individual values were first log transformed. The best correlation was obtained between α_p and α_e; the correlation coefficient, R_{pe} was found to be 0.68 (significant at the 99% confidence level and with a 95% confidence interval $0.27 \leq R_{pe} \leq 0.88$). The correlation coefficient, R_{pro}, between α_p and α_{ro} was somewhat smaller, namely, 0.58 (significant at the 95% confidence level, with a 95% confidence interval $0.11 \leq R_{pro} \leq 0.83$). The lowest correlation was found between α_p and α_r, namely, $R_{pr} = 0.56$, with a similar confidence level as R_{pro}. These three correlation coefficients, namely, 0.68, 0.58, and 0.56 are all quite high with good confidence levels, for this type of data. In light of the limitations of the lumped approach on which (3) is based, these correlations can be considered significant. Therefore even for hilly watersheds, the adoption of the lumped approach (i.e., the use of basin scale parameter values to estimate an average α) can produce useful results.

The question remains now why the correlation coefficient between α_p and α_e is higher than between α_p and the other two, or why the morphological-soil parameters are better correlated with α_e (see Table 3) than with α_r or α_{ro}. The reaction factors α_e were calculated as the averages of the slopes of the upper and lower envelopes. As found in the section Linearity of Lumped System, (see also Figure 3) the slope of the lower envelope remained fairly constant with duration of the dry period, whereas the slope of the upper envelope decreased continuously as the length of the dry period increased. Hence the upper envelope of all the data points reflects the recession behavior in the early stages of a dry period. This means that the α_e, which depend on the slope of this upper envelope, are also more heavily influenced by the early drainage behavior of the catchment, than the other two reaction factors α_{ro} and α_r; these were obtained by least squares, so that they characterize more the average long-term drainage behavior. Since the α_p were derived from morphologic and soil characteristics, the fact that R_{pe} is the largest suggests that these characteristics affect outflow behavior more strongly in the early phases of a dry period and that their effect decreases as the rainless period becomes longer. The fact that R_{pro} is the smallest of the three probably indicates that as already intimated earlier, the variations in the small values of E are, indeed, unreliable, and that more robust estimates of α can be obtained by means of (5) by simply constraining the intercept at $E = 0$. This issue will be discussed further in the following section.

Beside the correlation coefficients, a second measure of the quality of the predicted values of α is their magnitude. A direct comparison of the columns of α values given in Table 2 shows that the predicted values α_p are roughly two orders of magnitude smaller than the actual ones. In addition to deficiencies in the model leading to (3), the underprediction of the reaction factor by (3) can also be due to the fact that any of the variables K/f, $1/B$, or θ may have been taken too small. However, B does not vary much within a basin; also, the θ values were determined by a reliable method [see *Zecharias and Brutsaert*, 1985], and even large discrepancies in θ would not produce major variations in the slope term T. Therefore the cause for the underestimation of α may be the parameter K/f; actually, as shown in Table 3 it is also the parameter which is least correlated with α.

The hydraulic conductivities given in the Soil Survey reports are the result of laboratory permeability tests of soil samples [*Olson*, 1976, p. 32] and are therefore local parameter values. As such they probably represent only the effect of interstices on the water-transmitting properties of the aquifer material. Basin scale or overall hydraulic conductivities, on the other hand, include the effect of both micro- and macropores (root holes, worm holes, cracks, and fissures). Consequently, they are much larger than the corresponding local scale or laboratory values. Since the latter were used to compute α from (3), they may be the cause for most of the underprediction of the reaction factor values. In contrast, drainable porosity has a much smaller range of possible values and is therefore not significantly affected by the scale of measurement. However, the use in (3) of possibly overestimated f values (as obtained from *Johnson's* [1971] curves) may also have, to a lesser extent, contributed to the underprediction of α.

Groundwater Evaporation

The application of the least squares method to the data points resulted in E values (see Table 2) which vary widely from basin to basin and whose average value is considerably smaller than the known average value for the region [e.g. *Brutsaert*, 1982]. Differences in land use, such as proportion of forested to cleared or cultivated areas, would be expected to be reflected in differences of evaporation rates of the basins. However, most of the variation in the computed rates may be apparent rather than real. As mentioned, these E values are probably within the range of error of the discharge measurements. Therefore it is difficult to make a meaningful analysis of the variation in computed E.

The generally small magnitude of the individual E values, on the other hand, seems to suggest that very little of the overall watershed evaporation is furnished by groundwater. In any given watershed, the watertable tends to approach the land surface only in riparian zones along stream channels, which usually occupy a small fraction of basin area. Over the remainder of the watershed, there may be a decoupling of the upper unsaturated layers from the lower saturated zone of the unconfined aquifers. Under such conditions evapotranspiration is drawn mostly from upper zone soil moisture, and groundwater storage is affected much less by this depletion mechanism. Note that in a study of 23 km^2 basin in Alabama, *Daniel* [1979] found that evaporation from groundwater was only about 0.16 mm/day. Even smaller values (about 0.04 mm/day) may be derived from the base flow data of *Tschinkel* [1963] obtained in southern California if E is referred to the entire watershed area and not just to the riparian woodlands. These findings obtained by different methods are consistent with the E values derived in the present work.

CONCLUDING REMARKS

In an earlier study [*Zecharias and Brutsaert*, this issue] it was determined that drainage density, average basin slope, and length of perennial streams are the most important geomorphologic controls of groundwater discharge. These findings are confirmed herein, since the first two of these parameters appear in the expression for α (3). The physical model used here represents groundwater outflow q from an individual cross section of a hillslope into the adjacent stream channel. Therefore the third parameter (the total length of perennial streams) enters in the computation through the integration process of the elemental outflows to derive the total outflow Q at the outlet of the basin.

The outflow equation (2) used in this study is of the exponential decay form and is therefore equivalent to the lumped storage model formalized by *Sugawara and Maruyama* [1956], *Nash* [1958], *Dooge* [1973] and others. Evidently, the notion that groundwater outflow from a watershed can be characterized by an exponential decay function can be traced to the work of *Boussinesq* [1877], and over the years it has been applied widely in hydrology with considerable success. In the present paper this outflow equation is derived on the basis of a simple conceptual model by means of hydraulic theory of groundwater and in terms of basin morphologic and soil characteristics.

Because the physical characteristics (mainly slope and, to a lesser extent, drainage density and hydraulic conductivity) are not uniformly distributed within the basins in the region, a distributed model approach may perhaps yield better results. However, the primary advantage of adopting an average value of the reaction factor α, that is the lumped linear approach with basin scale parameters, is its convenience and practical applicability.

The values of the geomorphologic variables derived from maps, namely the slope θ and the drainage density $(1/2B)$ appear to be suitable for the parameterization of base flow. In contrast, the values of the soil variable (K/f) obtained from County Soil Survey reports are poorly coordinated with α and probably around two orders of magnitude too small. This discrepancy suggests that the local or small-scale estimation of some parameters does not fully capture all features of the phenomena involved at larger scales. In the case of hydraulic conductivity, this is probably due to the presence of meso- or macropores in the aquifers. More research is required to bring clarity in this problem.

APPENDIX: CONCISE PARAMETERIZATION OF HILLSLOPE DRAINAGE

Quasi Steady State Approach

Equation (1), first derived by *Boussinesq* [1877], embodies the so-called hydraulic theory of groundwater. The capillary flow above the water table is neglected, so that the latter can be taken as a true free surface; it is also assumed that the Dupuit (-Forchheimer) assumptions can be adapted to the sloping aquifer. In addition, in most applications the aquifer material is taken as homogeneous and isotropic.

In the quasi steady state approach it is further assumed that the shape of the moving water table is the same as that calculated for steady flow conditions. Under steady conditions the outflow from a slab is equal to the recharge. If i $[L/T]$ is the recharge rate, Figure 2 shows that at any x one has to a good approximation (since $D < B$) by virtue of (1)

$$Kh(dh/dx + \tan\theta) = i(B/\cos\theta) - x) \qquad \text{(A1)}$$

Upon linearization by putting $\bar{h} = h$, with the boundary conditions $h = h_1$ for $x = 0$, and $h = h_2$ for $x = B/\cos\theta$, integration of (A1) yields

$$i = 2K\bar{h}[(h_2 - h_1)\cos^2\theta + B\sin\theta]/B^2 \qquad \text{(A2)}$$

For steady flow conditions $(iB/\cos\theta) = q[L^2/T]$ is the outflow rate from the aquifer per unit length of channel. In the present derivation it is assumed that the channel is empty, so that $h_1 = 0$. Also, to keep the flow system linear it is assumed that h_2 is a constant fraction p of the aquifer depth D such that $0 \le p \le 1$. Thus (A2) can be written as

$$q = \alpha S \qquad \text{(A3)}$$

where α is the aquifer response constant given by (3); S $[L^2]$ is the amount of water stored (per unit length of channel) in the aquifer, given by

$$S = f\bar{h}B/\cos\theta \qquad \text{(A4)}$$

in which f is the drainable porosity. The unsteady flow problem can now be solved by means of the lumped equation of continuity

$$q + EB = -dS/dt \qquad \text{(A5)}$$

where $E[L/T]$ is the mean evapotranspiration rate. Integration of (A5) with (A3) produces the desired result (2).

Comparison With Other Studies

The parameterization used in the present study was also compared with the experimental results of *Hewlett and Hibbert* [1963]. Their data include $q_0 = 0.692$ m^3/day, $K = 168$ mm/hour, $\theta = 21.8°$, $B/\cos\theta = 13.72$ m, $D = 0.91$ m, total outflow $V_w = 1.26$ m^3, and total volume of soil drained $V_s = 10.58$ m^3. The drainable porosity f is determined by dividing V_w by V_s which gives a value of 0.12. In the present study, some of these parameters had to be adjusted to ensure that the assumptions made in developing the physical model are approximately satisfied by the experimental setup.

To account for the small wedge of soil at the lower end of the trough, which remained undrained at the end of the experiment, the effective height of the trough was taken as 0.86 m. For the same reason, the flow direction had an inclination of 21.8° only in the upper part of the trough; the general direction of flow at the lower end was horizontal. Therefore to obtain an "average" inclination that is applicable over the entire length of the trough, the given angle was reduced by a third as suggested by the geometry of the structure. It is known that fitting free surface models to field drainage problems requires an adjustment of the hydraulic conductivity. The results of *El-Kadi and Brutsaert* [1986] for outflow from unconfined aquifers in which B/D is of the order of 10, suggest

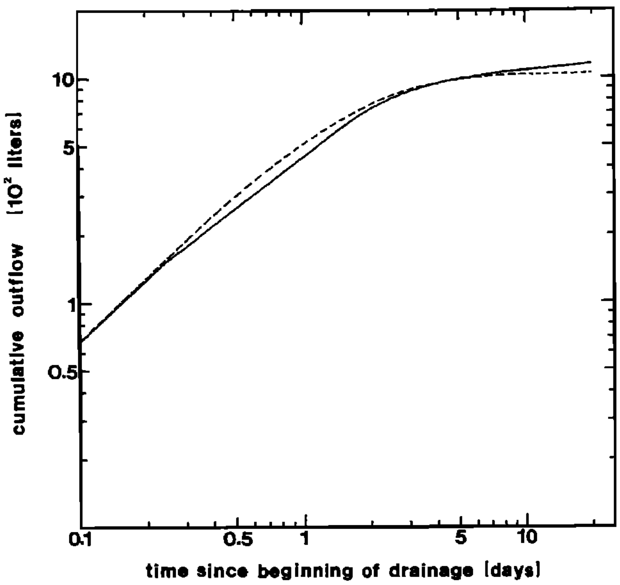

Fig. 4. Comparison of outflow volumes computed with (2) and (3) of quasi steady state approach (dashed curve) with experimental results of *Hewlett and Hibbert* [1963] (solid curve).

that a reduction factor of 0.5 is probably appropriate; accordingly, a value of $K = 84$ mm/hour was taken. The linearization parameter p was taken as 1/3; this value is suggested by comparison of *Polubarinova-Kochina*'s [1962, p. 507] exact solution of the nonlinear Boussinesq equation for drainage from an infinitely long aquifer with Edelman's approximate solution obtainable by linearization (as quoted by *Kraijenhoff van de Leur* [1979] and *Brutsaert and Nieber*, [1977]).

The use of these parameters in (3) gave an α value of 0.67, which was, in turn, used in (2) to compute the cumulative outflow volumes for $E = 0$. The results are presented in Figure 4, where they can be compared with the experimental results of *Hewlett and Hibbert* [1963]. The agreement is better than, or at least as good as, that for any of the other models analyzed by *Sloan and Moore* [1984] and *Stagnitti et al.* [1986].

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