A Multi-Scale Three-Dimensional Variational Data Assimilation Scheme and Its Application to Coastal Oceans

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3DVAR Data Assimilation and Forecast Cycle

- Diurnal variation
- Rapid response to wind stresses
- Eddies, fronts, filaments, etc

\[ x^a = x^f + \delta x \]

Initial condition

6-hour forecast

6-hour assimilation cycle

3-day forecast

Aug.1 00Z  Aug.1 06Z  Aug.1 12Z  Aug.1 18Z  Aug.2 00Z

Time
Autonomous Ocean Sampling Network (AOSN) Experiment August, 2003

“Bring together sophisticated new robotic vehicles with advanced ocean models to improve our ability to observe and predict the ocean”

Three Level Nested Monterey Bay ROMS Model SST Shaded Relieved with SSH

9 km  3km  1 km

www.mbari.org/aosn
An Incremental There-Dimensional Variational Data Assimilation (3DVAR)

\[
\begin{align*}
\min_x J(x) &= \frac{1}{2} (x - x^f)^T B^{-1} (x - x^f) + \frac{1}{2} (Hx - y)^T R^{-1} (Hx - y) \\
\min_x J(\delta x) &= \frac{1}{2} \delta x^T B^{-1} \delta x + \frac{1}{2} (H\delta x - \delta y)^T R^{-1} (H\delta x - \delta y) \\
\delta y &= y - Hx^f
\end{align*}
\]

1. Real-time capability

2. Implementation with sophisticated and high resolution model configurations

3. Flexibility to assimilate various observation simultaneously

(Li et al., 2006, MWR; Li et al., 2008, JGR)
AOSN Intensive Observations

- T/S profiles from gliders
- Ship CTD profiles
- Aircraft SSTs
- AUV sections
- HF radar velocities

Glider and AUV tracks

Ships, Aircrafts, and HF radars

T/S Profile Data
Performance of ROMS3DVAR
August 2003

Comparison of Glider-Derived Currents
(vertically integrated current)

(Chao and Li et al., 2009, DSR)
Southern California Coastal Ocean Observing System (SCCOOS)

Challenge: Assimilating sparse vertical profiles along with high resolution observations for a very high resolution model

Decorrelation length scales: 15-50km
Challenges with 3DVAR
Fourier Series Expansion of Homogenous Errors

\[ e(x) = \sum_{n=\infty}^{\infty} e_n \exp(inx) \]

\[ e_n = \frac{1}{2\pi} \int_{-\infty}^{\infty} e(x) \exp(-inx) dx \]

\[ \langle e_m e_n^* \rangle = \begin{cases} 
0, & m \neq n \\
 c_n, & m = n 
\end{cases} \]
Wiener-Khintchine Theorem

Error Covariance

\[ c(r) = \langle e(x)e(x + r) \rangle \]

\[ c_n = \frac{1}{2\pi} \int_{-\infty}^{\infty} c(r) \exp(-i\pi r) dr \]

\[ c(r) = \int_{-\infty}^{\infty} c_n \exp(i\pi r) dn \]

\[ c_n = e_n^2 \quad \text{power spectral density} \]
Error Covariance in 3DVAR: Smoothing and Spreading

\[ c(r) = e^{-\frac{r^2}{2D^2}} \]

\[ c_n = \frac{D}{\sqrt{2\pi}} \exp\left(-\frac{n^2 D^2}{2}\right) \]

\[ \exp(-\frac{r^2}{2D^2}) \]
A Multi-Decorrelation Length Scale Scheme for High Resolution Models?

Background Error

\[
x = x_L + x_S
\]

\[
e = e_L + e_S
\]

\[
\langle e_L e_S^T \rangle = 0
\]

\[
B = B_L + B_S
\]
3DVAR with a Background Error Covariance of Multi-Decorrelation Length Scales

\[ \min_{\delta x} J(\delta x) = \frac{1}{2} \delta x^T (B_L + B_S)^{-1} \delta x + \frac{1}{2} (H \delta x - \delta y)^T R^{-1} (H \delta x - \delta y) \]

\[ \min_{\delta x_L} J(\delta x_L) = \frac{1}{2} \delta x_L^T B_L^{-1} \delta x_L + \frac{1}{2} (H \delta x_L - \delta y)^T (H B_S H^T + R)^{-1} (H \delta x_L - \delta y) \]

\[ \min_{\delta x_S} J(\delta x_S) = \frac{1}{2} \delta x_S^T B_S^{-1} \delta x_S + \frac{1}{2} (H \delta x_S - \delta y)^T (H B_L H^T + R)^{-1} (H \delta x_S - \delta y) \]

\[ p(x_L \mid y) \]

\[ p(x_S \mid y) \]

(Lorenc, 1986)

(Li et al., 2011, QJRMS, in revision)
Multi-Scale Representativeness Errors

Observational Error Covariance for Large Scale

\[ R + H B_S H^T \]

\[
e^o_L = \delta y - H \delta x^t_L
= (y - y^t) - (H x^t - y^t) - H (x^b_S - x^t_S)
= e^{om} + e^{or} + e^{or}_S
\]

Measurement error + Representativeness error + Multi-scale representativeness error
Multi-Scale Data Assimilation

High resolution Observation

\[ y^h = y^h_L + y^h_S \]

Multi-scale DA

\[
\begin{align*}
\min_{\delta x_L} J(\delta x_L) &= \frac{1}{2} \delta x_L^T B_L^{-1} \delta x_L + \frac{1}{2} (H \delta x_L - \delta y^h_L)^T R_L^{-1} (H \delta x_L - \delta y^h_L) \\
\min_{\delta x_S} J(\delta x_S) &= \frac{1}{2} \delta x_S^T B_S^{-1} \delta x_S + \frac{1}{2} (H \delta x_S - \delta y^h_S)^T R_S^{-1} (H \delta x_S - \delta y^h_S)
\end{align*}
\]
3DVAR Formulations

3DVAR
\[
\min_x J = \frac{1}{2} (x - x^f)^T B^{-1} (x - x^f) + \frac{1}{2} (Hx - y)^T R^{-1} (Hx - y)
\]

AB-3DVAR
\[
\begin{align*}
\min_{\delta x_L} J &= \frac{1}{2} \delta x_L^T B_L^{-1} \delta x_L + \frac{1}{2} (H \delta x_L - \delta y)^T \left( H B_S H^T + R \right)^{-1} (H \delta x_L - \delta y) \\
\min_{\delta x_S} J &= \frac{1}{2} \delta x_S^T B_S^{-1} \delta x_S + \frac{1}{2} (H \delta x_S - \delta y)^T \left( H B_L H^T + R \right)^{-1} (H \delta x_S - \delta y)
\end{align*}
\]

MS-3DVAR
\[
\begin{align*}
\min_{\delta x_L} J &= \frac{1}{2} \delta x_L^T B_L^{-1} \delta x_L + \frac{1}{2} (H \delta x_L - \delta y_L^h)^T R_L^{-1} (H \delta x_L - \delta y_L^h) \\
\min_{\delta x_S} J &= \frac{1}{2} \delta x_S^T B_S^{-1} \delta x_S + \frac{1}{2} (H \delta x_S - \delta y_S^h)^T R_S^{-1} (H \delta x_S - \delta y_S^h)
\end{align*}
\]
Experiments with Idealized Problems

**True State**

\[ x_n^t = S_0 \sum_{k=1}^{K} a_k^t \cos\left(\frac{k \pi n}{N} + \phi_k^t\right) \]

\[ a_k^t = k^\gamma \]

\[ \phi_k^t = \alpha_k \pi, \alpha_k \in (-1,1) \]

\[ N = 200, K = N / 5 \]

**Background/First Guess**

\[ x_n^b = S_0 \sum_{k=1}^{K} a_k^b \cos\left(\frac{k \pi n}{N} + \phi_k^t\right) \]

\[ a_k^b = \beta^k a_k^t \]

**Observations**

\[ y^o = x_n^t + a_e^o e^o \]
Difference between SD-3DVAR, MD-3DVAR, and MS-3DVAR Solutions

Patchy Observation

\[ B_{ij} = b_i^2 \exp\left[-\frac{(i-j)^2}{2D^2}\right] \]
MS-3DVAR Work Flow

Forecast $\chi^f$

Observation innovation $\delta y$

$x^f_L, x^f_S$

$B_L, B_S$

LS-3DVAR

SS-3DVAR

Increment $\delta x^a_L$

Increment $\delta x^a_S$

$x^a_L = x^f_L + \delta x^a_L$

$x^a = x^f + \delta x^a_S$

\[
\begin{align*}
\min_{\delta x_L} J &= \frac{1}{2} \delta x_L^T B^{-1}_L \delta x_L + \frac{1}{2} (H \delta x_L - \delta y)^T (HB_S H^T + R)^{-1} (H \delta x_L - \delta y) \\
&\quad + \frac{1}{2} (H \delta x_L - \delta y^b_L)^T R^{-1}_L (H \delta x_L - \delta y^b_L) \\
\min_{\delta x_S} J &= \frac{1}{2} \delta x_S^T B^{-1}_S \delta x_S + \frac{1}{2} (H \delta x_S - \delta y)^T (HB_L H^T + R)^{-1} (H \delta x_S - \delta y) \\
&\quad + \frac{1}{2} (H \delta x_S - \delta y^b_S)^T R^{-1}_S (H \delta x_S - \delta y^b_S)
\end{align*}
\]
Kronecker Product Formulation of 3D Error Correlations

“NMC” Method: 48h-24h Forecast

\[
B = \Sigma C \Sigma
\]

\[
C^{\xi \eta \kappa} = C^{\xi \kappa} \otimes C^{\eta}
\]

\[
C^{\alpha \xi \eta \kappa} = G^{\xi \kappa} \left( G^{\xi \kappa} \right)^T \otimes G^{\eta} \left( G^{\eta} \right)^T
\]

\[
= \left( G^{\xi \kappa} \otimes G^{\eta} \right) \left( G^{\xi \kappa} \otimes G^{\eta} \right)^T
\]
Improved Performance with SCCOOS
MS-3DVAR Performance
1989 Exxon Valdez Supertanker Oil Spill in the Prince William Sound

Struggling sea lion during the tragedy days

Prince William Sound

Blind sea lion present day
Field Experiment 2009 Prediction of Drifter Trajectories in the Prince William Sound

Oil Spill: 1989 Exxon Tanker Wreck, Prince William Sound, Alaska

L0  10km
L1  3.6km
L2  1.2km
Effective Assimilation of High Frequency Radar High Resolution Velocities during Field Experiment 2009

Surface Currents
HF radar observed (Red), ROMS (Black)

(Schoch and Chao, 2010, EOS)
Summary

• A multi-scale 3DVAR scheme with partitioned cost functions was developed
• MS-3dVAR used multi-decorrelation length scales to construct background error covariance
• Effectiveness of the assimilation of both sparse and high resolution observations was improved
  ▪ Observation oriented covariance
  ▪ Reduced representativeness errors