Using Adjoint Models to Understand the Response of the Ocean Circulation to the North Atlantic Oscillation

Kettyah Chhak, Georgia School Board
Andy Moore, UC Santa Cruz
Ralph Milliff, Colorado Research Associates
Timescales

• Rapid fluctuations in ocean surface forcing are:
  - considerable in amplitude
  - “fast” compared to ocean circulation
  - can be considered as stochastic in time
  - spatially coherent (storm tracks, standing waves)

• Ocean response – perturbation development:
  - linear nonmodal
  - linear modal
  - nonlinear
Perturbation Development

**Nonnormal circulation**

Linear nonmodal  \quad \text{Linear modal}  \quad \text{Nonlinear}

“Nonnormality enhances variance”, Ioannou (JAS, 1995) (BL, met, climate, ocean)

**Normal circulation**

Linear  \quad \text{Linear}  \quad \text{Nonlinear}
The North Atlantic Oscillation

- "climate" regime
- "weather" regime

Stochastic!
Timescales

• Rapid fluctuations in ocean surface forcing are:
  - considerable in amplitude
  - “fast” compared to ocean circulation
  - can be considered as stochastic in time
  - spatially coherent (storm tracks, standing waves)

• Ocean response – perturbation development:
  - linear nonmodal
  - linear modal
  - nonlinear
Considerable Interest!


What did we observe and how predictable is it?
Questions

• What does the stochastically-forced ocean variability look like?
• How do the stochastically-excited perturbations evolve?
• What is the net influence of the stochastically-forced variability on the ocean circulation?
• How effective is the NAO at exciting ocean variability?
Conclusions

• Stochastically forced variability can be as large as intrinsic variability.
• Nonmodal interference dominates perturbation growth during first 10-14 days.
• Significant deep circulations due to rectified topographic Rossby waves.
• NAO is optimal for inducing variance on subseasonal timescales.
• Chhak et al (2009, JPO, 39, 162-184.)
QG model (Milliff et al., 1996):
- 1/5 (zonal) × 1/6 (merid) degree resolution, 5 levels
- Wind stress derived from CCM3

![Annual Mean (61Sv, 1.4m/s)](image1)

![Bathymetry](image2)

![Perturbation Energy](image3)

- Stochastically Forced
- Intrinsic
Nonmodal Linear Behaviour

Initial structure of surface perturbation

Perturbation enstrophy undergoes nonmodal growth

Perturbation energy does not!
\[
\delta = 4\pi/5 \quad \sigma_1 = -0.05 \quad \sigma_2 = 10\sigma_1
\]

\[
E=1 \quad \frac{ds}{dt} = As
\]

\[
A = \begin{pmatrix}
\sigma_1 & (\sigma_2 - \sigma_1)\cot\delta \\
0 & \sigma_2
\end{pmatrix}
\]
Nonmodal Linear Behaviour

Transient growth of enstrophy (evidence for modal interference)

Unit amplitude NAO pert

Barotropic Rossby wave modes

$Q$

$E$

TRW modes

Resultant pert at later time

$\theta$

$t=0$

$\theta$

$t=10$

Barotropic Rossby wave modes

Nonmodal Linear Behaviour

Transient growth of enstrophy (evidence for modal interference)
Nonmodal Linear Behaviour

Transient growth of enstrophy (evidence for modal interference)

Unit amplitude NAO pert

Barotropic Rossby wave modes

Time (days)

TRW modes

Resultant pert at later time

Barotropic Rossby wave modes

$\psi^B = 0$, flat bottom

$\psi^B(t)$

Resultant pert at later time

TRW modes

Barotropic Rossby wave modes

$t=0$

$t=10$
\[ \overline{k_2} \]

Pert. Energy

(\text{Perturbation Enstrophy})

\text{(Mean squared wavenumber or radius of gyration)}
Time Evolution of $\delta E$, $\delta Q$, and $\bar{k}_2$
Linear Behaviour via Ensemble Methods

100 member, 30 day ensembles forced by different wintertime realizations of the NAO

Deep ocean structure of 1st EOF of Enstrophy (~87%)
Nonlinear Behaviour

Rectified deep wintertime circulation due to Topographic Rossby Waves ~ 2Sv

(Also noted by McWilliams, 1974; Willebrand et al, 1980)
Ensemble variance:

\[ V(t) = \left\langle \xi'^T B \xi' \right\rangle = \sigma^2 \sum_{i=1}^{N} \lambda_i \left( e^T q_i \right)^2 \]
Observability and Stochastic Optimals (SOs)

Entire forcing space Dimension $\sim 10^5$

For $T\sim 10-90$ days, 1$^{st}$ Stochastic Optimal accounts for $\sim 65\%$ of variance
Stochastic Optimals

Energy SO #1

Enstrophy SO #1

Energy SO #2

Ekman Pumping Velocity

Enstrophy SO #2
Conclusions

- Stochastically forced variability can be as large as intrinsic variability.
- Nonmodal interference dominates perturbation growth during first 10-14 days.
- Significant deep circulations due to rectified topographic Rossby waves.
- NAO is optimal for inducing variance on subseasonal timescales.
- Chhak et al (2009, JPO, 39, 162-184.)
Comments

• Results applicable to stochastic forcing of ocean by other teleconnection patterns.
• Implications for interpretation of observations.
• Implications for ocean predictability.
Observability and Stochastic Optimals (SOs)

Entire forcing space Dimension $\sim 10^5$

For $T\sim10$-90 days, 1$^{st}$ Stochastic Optimal accounts for $\sim65\%$ of variance
Observability and Stochastic Optimals (SOs)

Entire forcing space Dimension $\sim 10^5$

<table>
<thead>
<tr>
<th>$T$ (days)</th>
<th>NAO variance explained by 1st SO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q</td>
</tr>
<tr>
<td>10</td>
<td>$\sim 67%$</td>
</tr>
<tr>
<td>20</td>
<td>$\sim 67%$</td>
</tr>
<tr>
<td>30</td>
<td>$\sim 67%$</td>
</tr>
<tr>
<td>60</td>
<td>$\sim 65%$</td>
</tr>
<tr>
<td>90</td>
<td>$\sim 65%$</td>
</tr>
</tbody>
</table>