Ensemble Data Assimilation:
Perturbing the background state to represent model uncertainties

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Ensemble Data Assimilation:
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$$x_a = K y_q + (I_q - KH) x_b$$

- EnDA perturbing \( y \) and \( x_b \)
- Comparisons with EnDA with different model error representation and EnDA where only data error is represented
- Diagnostics on the B derived from all different EnDA
- EnDAs performance in the EPS
- Conclusion
Ensemble Data Assimilation
perturbing the background state to represent model uncertainties

Control Analysis

Member 1  
\[ y \pm \varepsilon^1_o \]

Member 2  
\[ x_b \pm \varepsilon^1_b \\
 y \pm \varepsilon^1_o \]

Member 3  
\[ y \pm \varepsilon^2_o \]

Member 10  
\[ \varepsilon_o \text{ has the magnitude of } \sigma_o \]
\[ \varepsilon_b \text{ has the magnitude of } \sigma_b \]
and the structure of B
Ensemble Data Assimilation: perturbing the background state to represent model uncertainties

\[ \sigma^2(t) \rightarrow \sigma_B^2(t), \sigma_R^2(t) \]
\[ \sigma^2(t+) \rightarrow \sigma_B^2(t+), \sigma_R^2(t+) \]

\[ \sigma_B^2(t+) = \sigma_Q^2(t+) + L\sigma_A^2(t)L^T = \sigma_Q^2(t+) + L\left( \frac{1}{\sigma_B^2(t)} + \frac{1}{\sigma_R^2(t)} \right)^{-1} L^T \]

\[ \sigma^2(t+) \rightarrow \left[ \sigma_Q^2(t+) + L\left( \frac{1}{\sigma_B^2(t)} + \frac{1}{\sigma_R^2(t)} \right)^{-1} L^T, \sigma_R^2(t+) \right] \]

\[ \sigma^2(t) \rightarrow \sigma_Q^2(t), \sigma_R^2(t) \]
\[ \sigma^2(t+) \rightarrow \sigma_Q^2(t+), \sigma_R^2(t+) \]

To be compared with
Ensemble Data Assimilation
Experiment set-up

Realization: 10 member
Resolution: T399T159L91
Period: 20081005-20081115

Model error representation:

- **BS** Spectral Stochastic Kinetic Energy Backscatter scheme (Berner et al. 2009)
- **ST** Stochastic representation of model error associated to parametrized physical processes tendencies (Buizza et al. 1999)
- **PX_b** Perturbed background with gaussian random correlated perturbation
- **O** Perturbed observation with gaussian random perturbation
- **OInfl** Perturbed observation with gaussian random perturbation and inflated background error variances

Systematic kinetic energy loss → numerical integrations and parametrization
Ensemble Data Assimilation: spread U L10

PX\textsubscript{b}, ST, BS, OInfl, O
Ensemble Data Assimilation: spread U L78
Ensemble Data Assimilation: spread T L10

PXb

ST

BS

OInfl

O
Ensemble Data Assimilation: AMSUA ch 6 Desroziers et al. 2005

\[ HBH = E(d_b^a(d_b^o)^T) \]
\[ d_b^a = Hx_a - Hx_b \]
\[ d_b^o = y - Hx_b \]

ST

BS

PX_b
Ensemble Data Assimilation: AMV >700 hPa

Desroziers et al. 2005

\[ HBH = E(d_b^a(d_b^o)^T) \]

\[ d_b^a = Hx_a - Hx_b \]

\[ d_b^o = y - Hx_b \]
EnDA: Observation Influence  AMSUA  ch6  Cardinali et al. 2004

\[
\frac{\partial H_x}{\partial y} = K^T H^T \quad K = (B^{-1} + H^T R^{-1} H)^{-1} H^T R^{-1}
\]
1. EDA[\text{PX}_b], EDA[\text{ST}], EDA[\text{BS}], EDA[\text{ST,BS}]: STD/EM

Roberto Buizza

In terms of T850, EDA[\text{PX}_b] has the largest spread and EDA[\text{BS}] the smallest for the whole forecast range. Adding BS to ST has a negligible impact.

In terms of rmse of the ensemble-mean, EDA[\text{PX}_b] and EDA[\text{ST}] have similar scores, both lower than EDA[\text{BS}] over NH in the medium-range, and over the tropics from ~ day 4.
1. EDA[PXₚ], EDA[ST], EDA[BS], EDA[ST,BS]: RPSS

Roberto Buizza

In terms of RPSS for T850, EDA[BS] has the lowest scores. EDA[PXₚ], EDA[ST] and EDA[ST,BS] have very similar scores, better over both NH and the tropics. This is probably a consequence of the better-tuned ensemble spread.
The ignorance score, which is more sensitive to the tail of the forecast probability distribution function, shows more differences between the experiments. In terms of IGN for T850, EDA[BS] has the lowest scores, followed by EDA[ST] and EDA[ST,BS], with EDA[PX_b] showing the best results over both NH and the tropics.
Perturbing the background state versus Others: Preliminary Conclusion

*Perturbing the background state add more spread in the tropics and extra-tropics*

- The increase of spread is observed in areas where the model is known to be wrong

- The increase of spread is linked with the dynamic activity

*Very easy to maintain does not require tuning from one model-cycle to an other*

*The diagnostic performed on the B matrix computed from different EnDA shows NO differences*

- Need of further investigation on the B matrix computation (Derber et Bouttier 1998), in particular to the applied balance operator

*Preliminary results from EPS show larger spread in the Tropics and in the Extra-Tropics*