Ensemble-based approximation of observation impact using an observation-based verification metric

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Research question

In a complex systems of observations, data assimilation and forecasts...

- How much do the individual observations contribute to the forecast quality?
- Are the observations used in an optimal way?

Motivation

The assessment of observation impact can help...

- to improve the interaction of observations, data assimilation and model
- to exclude data that systematically degrades the forecast.

Methods to determine observation impact

- Data-denial-experiments: Big computational cost
- Adjoint-based methods: Not available for all models, e. g. COSMO
- Ensemble-based methods Kalnay et al. [2012], Liu and Kalnay [2008], Sommer and Weissmann [2014]
Observation impact: Definition

Data denial impact of observations $d'$ relative to all observations $d$

$$J(d') = |e^d_f|^2 - |e^d_f - d'|^2$$

$d$: All available observations

$d'$: Small subset of observations whose impact one is interested in

Algorithm

Example

(a) $|e^d_f|^2$

(b) $|e^d_f - d'|^2$

(c) $J = \frac{1}{2} \left( |e^d_f|^2 - |e^d_f - d'|^2 \right)$
Approximation with ensemble perturbations

**LETKF update equation**

\[ \bar{x}_{aj} = x_{bj} \tilde{P}_a(j) Y_b^T R^{-1}(j)(y_o - \bar{y}_b) + \bar{x}_{bj} \]

**Variables**

- \( j \): Grid point
- \( \bar{x}_a \): Analysis mean
- \( X_b \): Background ensemble
- \( \tilde{P}_a \): Ensemble analysis error covariance matrix
- \( W^a(j) = \left( (K - 1)\tilde{P}_a(j) \right)^{\frac{1}{2}} \): Weight matrix
- \( Y_b \): Background ensemble in observation space
- \( R \): Observation error covariance matrix
- \( d = y_o - \bar{y}_b \): Observational increment
- \( \bar{x}_b \): Background mean
Approximation with ensemble perturbations

**LETKF update equation**

\[
\bar{x}_{aj} = X_{bj} \tilde{P}_a(j) Y_b \top R^{-1}(j)(y_o - \bar{y}_b) + \bar{x}_{bj}
\]

**Data denial observation impact**

\[
J(d') = |e^d_f|^2 - |e^{d-d'}_f|^2 = (e^d_f + e^{d-d'}_f) \cdot (e^d_f - e^{d-d'}_f)
\]

**Direct derivation [Kalnay et al., 2012]**

\[
e^d_f - e^0_f = \bar{x}^d_f - \bar{x}^0_f \approx \frac{1}{K-1} X^d_f (Y_b W^d) \top R^{-1} d
\]

\[
\Rightarrow J(d')
\]

\[
= (e^d_f + e^{d-d'}_f) \cdot (e^d_f - e^0_f - (e^{d-d'}_f - e^0_f))
\]

\[
\approx (e^d_f + e^{d-d'}_f) \cdot \left( \frac{1}{K-1} X^d_f (Y_b W^d) \top R^{-1} d' \right)
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\]

\[
\Rightarrow J(d') = 2e^d_f \cdot \left( - \frac{d}{dd'} \bigg|_{d'=0} e^{d-d'}_f \right) d' + O \left( |d'|^2 \right)
\]

**Taylor expansion [Sommer and Weissmann, 2015]**

\[
J(d') = J(0) + \frac{d}{dd'} \bigg|_{d'=0} J(d') d' + O \left( |d'|^2 \right)
\]

\[
= 2e^d_f \cdot \left( - \frac{d}{dd'} \bigg|_{d'=0} e^{d-d'}_f \right) d' + O \left( |d'|^2 \right)
\]

\[
= 2e^d_f \cdot \left( \frac{d}{dd'} \bigg|_{d'=d} \bar{x}^{d'}_f \right) d' + O \left( |d'|^2 \right)
\]

\[
\approx 2e^d_f \cdot \left( \frac{1}{K-1} X^d_f (Y_b W^d)^\top R^{-1} d' \right)
\]
Verification with...

\[ e_f = \overline{x_f} - x_a \]

\[ |e_f|^2 = \sum_{\text{gridpoints}} \frac{1}{2} (\overline{u}_f - \overline{u}_a)^2 + \frac{1}{2} (\overline{v}_f - \overline{v}_a)^2 \]

\[ \Rightarrow J(d') \approx 2e^d_f \cdot \left( \frac{1}{K-1} X^d_f (Y_b W^d)^T R^{-1} d' \right) \]

+ Homogeneous in space and time
- Strongly correlated to forecast

\[ e_f = H(\overline{x_f}) - y_o \]

\[ |e_f|^2 = \sum_{\text{observations}} \left( \frac{H(\overline{x_f}) - y_o}{\sigma} \right)^2 \]

\[ \Rightarrow J(d') \approx 2e^d_f \cdot \left( \frac{1}{K-1} Y^d_f (Y_b W^d)^T R^{-1} d' \right) \]

+ Independent of forecast
+ Computationally easy
- Unobserved regions/variables may be ignored
DWD Convective-scale assimilation and forecasting systems

**Kilometer-scale Ensemble Data Assimilation (KENDA)**
- Localized Ensemble Transform Kalman Filter for use with COSMO-DE (in development)

**Consortium for Small-scale Modelling (COSMO)**
- Operational limited-area model of Deutscher Wetterdienst
- Grid point model of non-hydrostatic equations
- Horizontal resolution: 2.8 km; 50 vertical levels

*Figure: COSMO-DE domain (≈ 1300 km × 1200 km)*

**Experimental settings**
- Test period: 10 June 2012 12:00 UTC – 13 June 2012 15:00 UTC
- Initialization every 3 h
- Forecast length 6 h
- 40-members ensemble
- Observations used:
  - AIREP (Aircrafts): $U$, $V$, $T$
  - PROF (Wind profiler): $U$, $V$
  - SYNOP (Ground stations): $U$, $V$, $T$, $RH$
  - TEMP (Weather Balloons): $U$, $V$, $T$, $RH$
Number of assimilated observations per station

10 June 2012 12:00 UTC – 13 June 2012 15:00 UTC

Number of AIREP observations

Number of PROF observations

Number of SYNOP observations

Number of TEMP observations
Impact per observation type

### Total impact

- **Impact per observation type:**
  - AIREP: $-0.06$
  - PROF: $-0.04$
  - SYNOP: $-0.02$
  - TEMP: $0$

- **Number of observations:**
  - AIREP: $0$
  - PROF: $1$
  - SYNOP: $2$
  - TEMP: $\times 10^5$

- **Number of stations:**
  - AIREP: $0$
  - PROF: $500$
  - SYNOP: $1000$
  - TEMP: 

### Impact per observation

- **Impact per observation:**
  - AIREP: $-4 \times 10^{-7}$
  - PROF: $-2 \times 10^{-7}$
  - SYNOP: $0$
  - TEMP: $-4 \times 10^{-3}$

### Impact per station

- **Impact per station:**
  - AIREP: $0$
  - PROF: $0$
  - SYNOP: $0$
  - TEMP: $-4 \times 10^{-3}$
Impact per observation type

Total impact

Impact per observation

Number of observations

Number of stations

Impact per station

One wind profiler equivalents...
Distribution of impact values

Histogram of individual observations impact values

- **AIREP.** Ratio neg/pos: 0.516 : 0.484
- **PROF.** Ratio neg/pos: 0.51 : 0.49
- **SYNOP.** Ratio neg/pos: 0.521 : 0.479
- **TEMP.** Ratio neg/pos: 0.518 : 0.482

- Non-Gaussian distribution
- Ratio of negative to positive values ca. 52:48
- Width of distribution $\gg$ Mean
Distribution of impact values

Histogram of individual observations impact values

Transformation of x-axis

\[ J(d') = |e^d|^2 - |e^{d-d'}|^2 \]  \[ \rightarrow \]  \[ \hat{J}(d') = \text{sign}(J(d')) \sqrt{|J(d')|} \]
Distribution of impact values

Histogram of individual observations impact values

Transformation of x-axis

\[
J(d') = |e^d|^2 - |e^{d-d'}|^2 \quad \rightarrow \quad \hat{J}(d') = \text{sign}(J(d')) \sqrt{|J(d')|}
\]
Distribution of impact values

Histogram of individual observations impact values

- Different slopes of negative and positive impact values
- Mismatch with PROF observations
Distribution of impact values

Histogram of individual observations impact values

Probability distribution

\[ p(J) \sim e^{-\alpha \sqrt{J} + \beta} \Rightarrow \langle J \rangle = \int dJ Jp(J) = -\frac{2}{\alpha^4} e^{-\alpha \sqrt{J} + \beta} \left( 6 + 6\alpha \sqrt{J} + 3\alpha^2 J + \alpha^3 J^{3/2} \right) \]
Impact per observation type

Total impact

![Graph showing impact per observation type]

- Qualitative match between approximation and data denial impact
- Bad match between approximation and data denial impact
- Discrepancy between estimated and smoothed impact hints at insufficient sampling

Reliability indicator

<table>
<thead>
<tr>
<th></th>
<th>AIREP</th>
<th>PROF</th>
<th>SYNOP</th>
<th>TEMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unfitted impact</td>
<td>-0.0094</td>
<td>-0.0216</td>
<td>-0.0421</td>
<td>-0.0061</td>
</tr>
<tr>
<td>Fitted impact</td>
<td>-0.0101</td>
<td>-0.0090</td>
<td>-0.0433</td>
<td>-0.0055</td>
</tr>
<tr>
<td>Ratio</td>
<td>0.93</td>
<td>2.39</td>
<td>0.972</td>
<td>1.11</td>
</tr>
</tbody>
</table>

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Ensemble-based approximation of observation impact
Cumulative distribution function of observation impact from experiment (green) and fit (blue)

Extreme values contribute only little to total impact (except for PROF)
Temporal impact distribution

**Observation time vs. impact**

- Temporally homogeneous distributions (low dependency on forecast time)
- Extreme PROF values during precipitation event
Low specificity of regions with positive and negative impact
Impact per observed variable

Normalized with number of observations

- Generally large temperature impact
- Small SYNOP wind impact
- Anisotropy of wind components impact
Dependency on verification

Each observation group has the largest impact by verification with itself
Definition of suitable metric including radar and satellite observations

Weighted metric

\[ \tilde{J}^A_\alpha = \frac{\alpha_{\text{AIREP}}}{J^\text{TOTAL}_{\text{AIREP}}} J^A_{\text{AIREP}} + \frac{\alpha_{\text{PROF}}}{J^\text{TOTAL}_{\text{PROF}}} J^A_{\text{PROF}} + \frac{\alpha_{\text{SYNOP}}}{J^\text{TOTAL}_{\text{SYNOP}}} J^A_{\text{SYNOP}} + \frac{\alpha_{\text{TEMP}}}{J^\text{TOTAL}_{\text{TEMP}}} J^A_{\text{TEMP}} \]

<table>
<thead>
<tr>
<th>Verification norm</th>
<th>AIREP impact</th>
<th>PROF impact</th>
<th>SYNOP impact</th>
<th>TEMP impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_{25/25/25/25} )</td>
<td>23%</td>
<td>31%</td>
<td>32%</td>
<td>13%</td>
</tr>
<tr>
<td>( J_{30/30/30/10} )</td>
<td>25%</td>
<td>35%</td>
<td>31%</td>
<td>9%</td>
</tr>
<tr>
<td>( J_{PS} )</td>
<td>37%</td>
<td>-1%</td>
<td>49%</td>
<td>16%</td>
</tr>
</tbody>
</table>
Signal propagation of AIREP observations

Data denial

$t = 0h$

$t = 6h$
Signal propagation of AIREP observations

Approximation

$t = 0h$

$t = 6h$
Summary

Tool for an approximated assessment of observation impact in an LETKF

- Fast a posteriori estimation of observation impact in a combined analysis and forecasting system
- Modification for the use of observations as verification
- Reliability indication (→ long averaging needed for stable results)
- Limit the approximation to short forecast times because of
  - Linearisation
  - (Static) localization
- Results depend on verification metric

Outlook

- Assessment of impact of more complex observations (Satellites, radar)
- Longer experiment period and operational implementation (DWD)

Literature


