Coupled atmosphere-ocean data assimilation in the presence of model error

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Introduction

Coupled DA methods are being developed to initialise forecasts of the coupled atmosphere-ocean system. Accounting for the coupling within the DA allows for an initial state to be found which lies on the coupled model attractor. This reduces the chance of initialisation shocks.
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Coupled DA faces many challenges, here we focus on the specific problem of **model error** which may be amplified via the coupling.

Model error in the coupled system **restricts the window length** which can be used with 4D-Var to something shorter than the optimal window length in an uncoupled Ocean DA scheme.

As the **ocean is poorly observed**, this has the effect of potentially sacrificing the accuracy of the ocean analysis in order to provide a more balanced coupled analysis.
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Aim of work is to understand how different coupling strategies react as window length is extended and the model error becomes more significant.
Coupling strategies (recap)

Each method is based on incremental 4D-Var. In each case the first guess comes from a coupled forecast.

**Strongly coupled DA**: uses the coupled model in both the inner and outer loops.

**Weakly coupled DA**: uses the coupled model in the outer loop but the inner loop is uncoupled.

**Uncoupled DA**: uses the uncoupled models in both the outer and inner loops. BCs at the interface are prescribed externally.
Model error in 4D-Var

- In 4D-Var have the assumption that the background state, \( x^b \), and the observations, \( \hat{y} \), are consistent with

\[
\begin{align*}
  x^b &\sim N(x_0^t, B) \quad \text{and} \quad \hat{y} \sim N(\hat{y}^t, \hat{R}) \\
  \text{and} \quad \hat{y}^t &= \hat{H}(x_0^t)
\end{align*}
\]

(This generalised observation operator, \( \hat{H}(x_0^t) \), includes the dynamical model which evolves the initial state forward to the time of the observations)
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\[ x^b \sim N(x^t_0, B) \quad \text{and} \quad \hat{y} \sim N(\hat{y}^t, \hat{R}) \]

and

\[ \hat{y}^t = \hat{H}(x^t_0) \]

(This generalised observation operator, $\hat{H}(x^t_0)$, includes the dynamical model which evolves the initial state forward to the time of the observations)

• However if model error becomes significant then this last assumption breaks down, and instead:

\[ \hat{y}^t = \hat{H}^t(x^t_0) \]
\[ = \hat{H}(x^t_0) + \epsilon \hat{H} \quad \text{where} \quad \epsilon \hat{H} \in \mathbb{R}^{\hat{p} \times 1} \]
Model error in 4D-Var
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If the model error is unaccounted for it has the effect of increasing the analysis error covariances

\[ E[\epsilon^a(\epsilon^a)^T] = P_{nm}^a + KE[\hat{\epsilon}(\hat{\epsilon})^T]K^T \]

where \( P_{nm}^a \) is the analysis error covariance matrix when no model error is present.

where \( K \in \mathbb{R}^{n \times \hat{p}} \) is the Kalman gain matrix.
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And if the model error is biased then the analysis error will also be biased

\[ E[\epsilon^a] = KE[\hat{\epsilon}]. \]

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Model error in incremental 4D-Var

- The difference between the three coupling strategies is in the way they have implemented incremental 4D-Var.
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- In incremental 4D-Var use a TL approximation

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• The total error perceived \( \epsilon \hat{\mathbf{H}} = \epsilon \hat{\mathcal{H}} + \epsilon^{\text{TL}} \) for each of the coupling strategies becomes
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**Strongly coupled**

\[
\epsilon^\hat{H} = \hat{H}^t(x_0^t) - \hat{H}^c(x_0^g) - \hat{H}^c|_{x^g}(x_0^t - x_0^g)
\]

**Weakly coupled**

\[
\epsilon^\hat{H} = \hat{H}^t(x_0^t) - \hat{H}^c(x_0^g) - \hat{H}^{uc}|_{x^g}(x_0^t - x_0^g)
\]

**Uncoupled**

\[
\epsilon^\hat{H} = \hat{H}^t(x_0^t) - \hat{H}^{uc}(x_0^g) - \hat{H}^{uc}|_{x^g}(x_0^t - x_0^g)
\]
The model error is assumed to be complex and from multiple sources.

**Atmosphere**
Assimilation model has missing physics and a bias in the large scale forcing.

**Ocean**
Assimilation model has perturbed diffusion parameters.

Results are shown for the July 2014 case study for a point in the NW Pacific.
Model error in the atmosphere (no DA)

Figure: Model error over 2 days for atmospheric temperature

Uncoupled DA with a poor estimate of the BCs

Uncoupled DA with a good estimate of the BCs
Model error in the atmosphere (no DA)

Figure: Model error over 2 days for atmospheric temperature

Observation error standard deviation is 1K.

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Uncoupled DA with a good estimate of the BCs
Model error in the ocean (no DA)

Figure: Model error over 2 days for oceanic temperature

Uncoupled DA with a poor estimate of the BCs

Uncoupled DA with a good estimate of the BCs
Model error in the ocean (no DA)

Figure: Model error over 2 days for oceanic temperature
Observation error standard deviation is 0.1K.

Uncoupled DA with a poor estimate of the BCs

Uncoupled DA with a good estimate of the BCs
Absolute analysis error ($|x^a - x^t|$)

Figures: Absolute error in temperature at the initial time, in the atmosphere (left) and ocean (below).

- Atmospheric observations every 3 hours, ocean observations every 6 hours.
- Each level is observed.
- \(B\) is diagonal.
Forecast error in the atmosphere $(M_t(x_t) - M(x^a))$

Figure: Forecasts of atmospheric temperature using coupled model initialised using different analyses computed using a 2 day assimilation window
Forecast error in the atmosphere 

\( M^t(x^t) - M(x^a) \)

Figure: Forecasts of atmospheric temperature using coupled model initialised using different analyses computed using a 2 day assimilation window.
Forecast error in the ocean 
\((M^t(x^t) - M(x^a))\)

Figure: Forecasts of oceanic temperature using coupled model, initialised using different analyses computed using a 2 day assimilation window.
Forecast error in the ocean 

\( (M_t(x^t)-M(x^a)) \)

Figure: Forecasts of oceanic temperature using coupled model, initialised using different analyses computed using a 2 day assimilation window.
Summary - 2 day window length

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Conclusions

• The coupling of the atmosphere and ocean can amplify the presence of model error.
• The effect of model error on the analysis depends on the coupled DA scheme used.
• Strongly coupled DA has been shown to be able to provide an analysis consistent with the forecast model at the expense of the accuracy of the ocean analysis.
• To improve the utility of strongly coupled DA need to be able to account for model error in the assimilation to allow for the window length to be extended.

• We are currently developing a weak-constraint coupled DA scheme to estimate the model error within the atmosphere.
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Thank you for listening
Initialisation shock
Initialisation shock - reduced observations