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Invariant Imbedded T-Matrix Method for Axial Symmetric Hydrometeors with Extreme Aspect Ratios

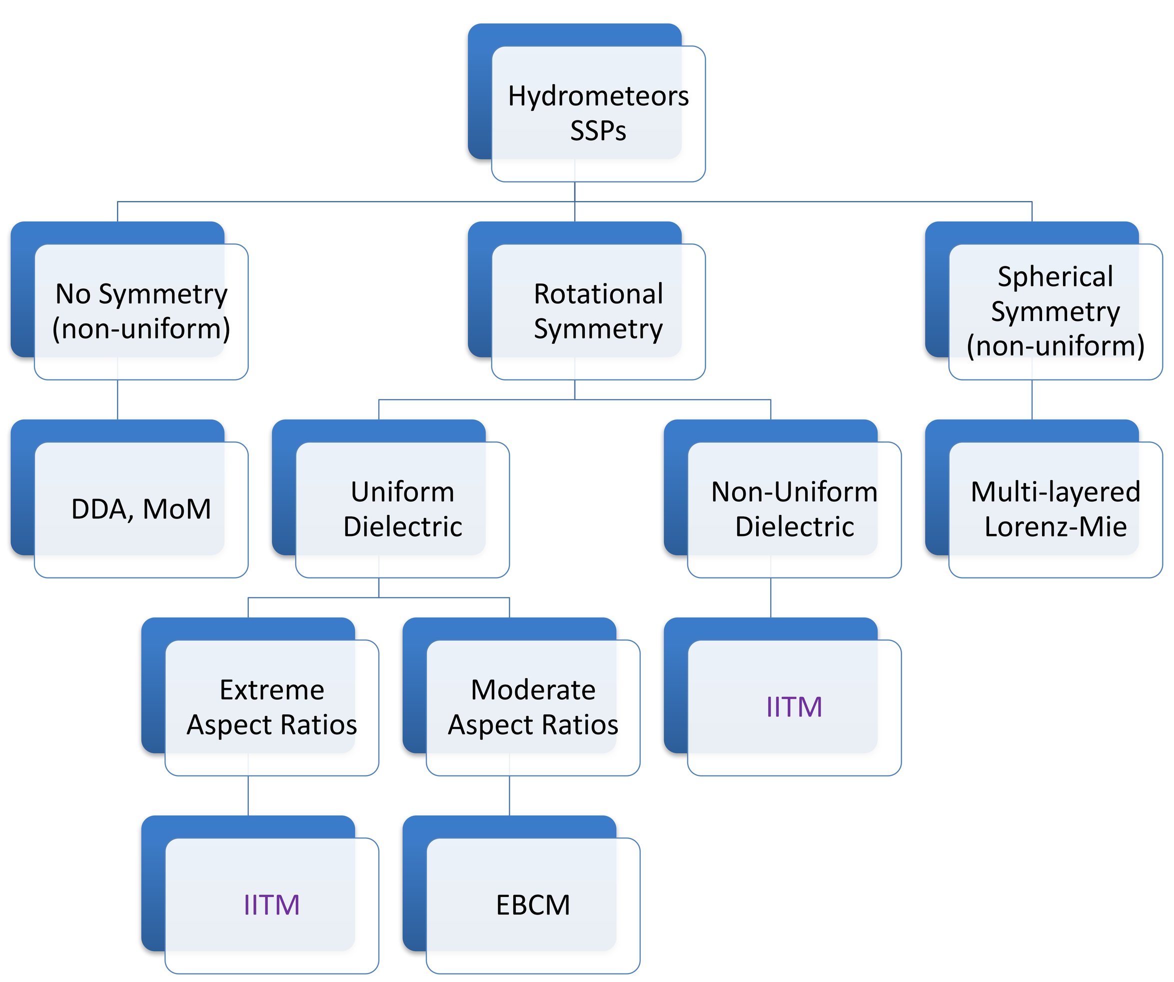
Craig Pelissier[1,3], Kwo-Sen Kuo[1,2], Tom Clune[1], Ian Adams[1], S. Joseph Munchak[1]
[1]NASA Goddard Space Flight Center, Maryland, USA; [2]Earth System Science Interdisciplinary Center, University of Maryland-College Park, Maryland, USA
[3]Science Systems and Applications Incorporation, Maryland, USA

Introduction

One of the most fundamental and crucial quantities for accurate precipitation retrievals is the single-scattering property (SSP) of each individual hydrometeor in the precipitating volume. Thus, an important step in minimizing retrieval uncertainty is to ensure that the particles used for single-scattering calculations resemble those that occur in nature, allowing the SSPs to be computed precisely with numerical techniques.

Hydrometeors with rotational symmetry (axial symmetry) are most efficiently computed using the T-Matrix methods. For hydrometeors with a uniform dielectric, the Extended Boundary Condition Method (EBCM) is the most efficient. However, for mixed-phase hydrometeors with non-uniform dielectrics EBCM is not applicable. In these cases, we can use the Invariant Imbedded T-Matrix (IITM), which we developed for this purpose. It is also noted that hydrometeors with more extreme aspect ratios can be computed with IITM than EBCM and that such hydrometeors are found in precipitating systems. For hydrometeors without any rotational symmetry, however, volume integral methods like Discrete Dipole Approximation (DDA) and the Method of Moments (MoM) must be employed.

Algorithm Flow-Chart for SSPs

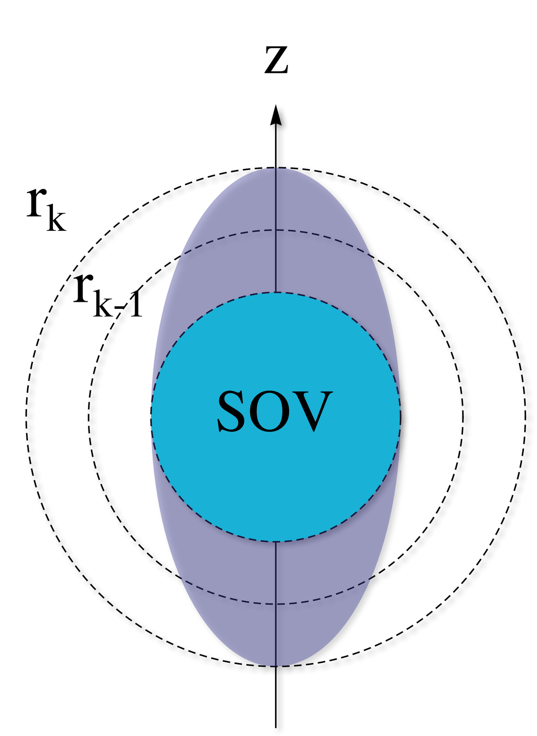


Implementation

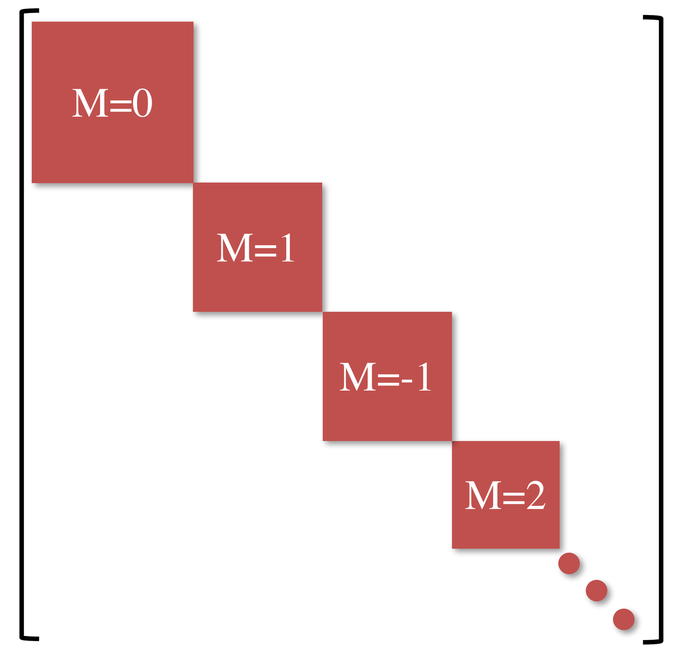
The IITM is typically an order of magnitude more computationally expensive than EBCM. To make IITM practical requires an efficient implementation that is optimized and takes advantage of parallelism in the IITM algorithm.

Algorithm Schematic

1. Initialize with Separation of Variables (SOV).
2. Perform quadrature over spherical shell r_{k-1} --- Gauss-Legendre quadrature.
3. Using recursion relations to get the T-Matrix at rk, $T(r_k)$. --- matrix inversion + linear algebra. Romberg integration used.

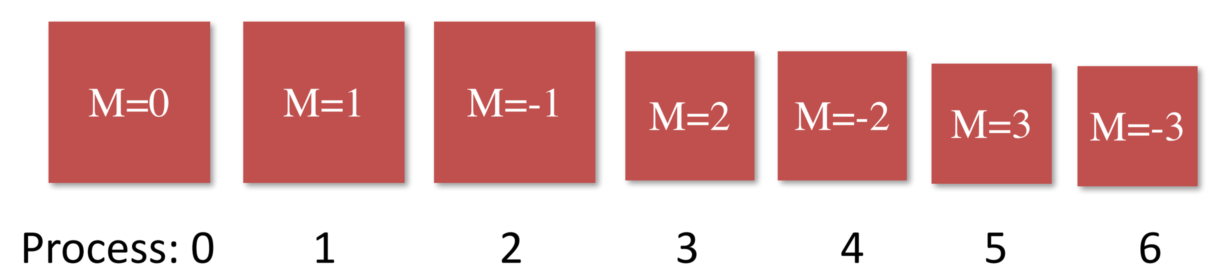


Block Structure/Parallelism



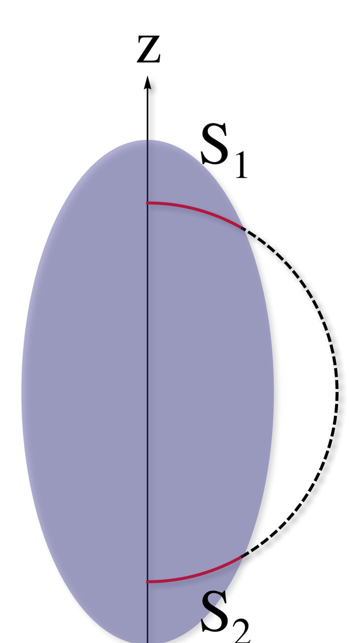
Rotational symmetry decouples "m"

$$T_{l'l,m'm} = T_{l'lm}\,\delta_{m'm}$$



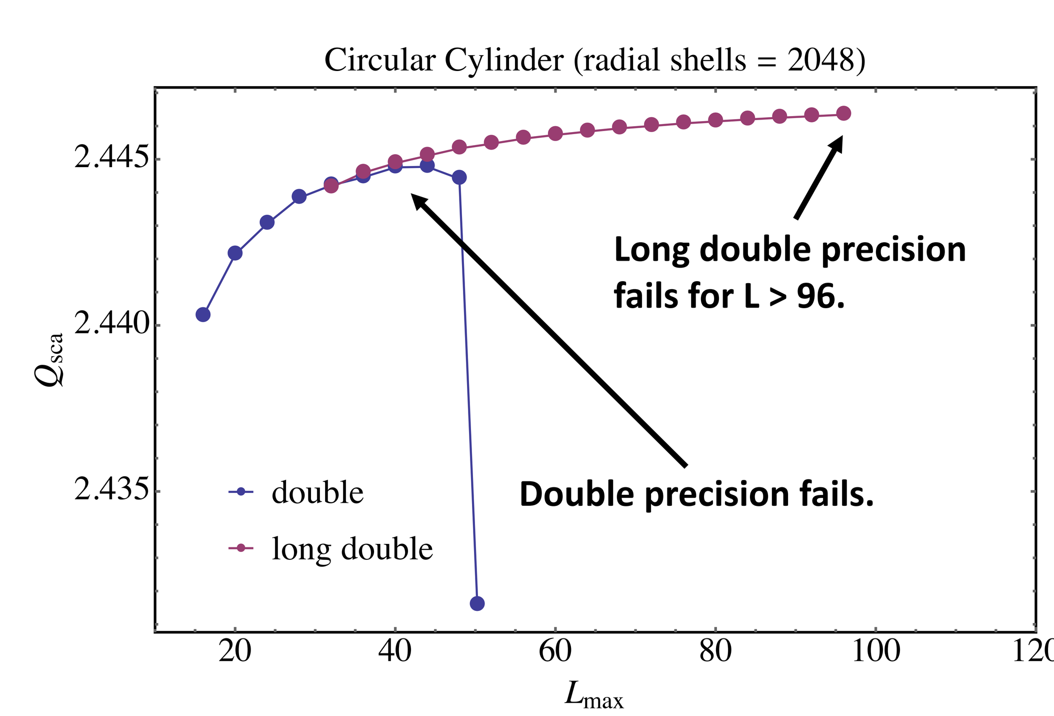
Naïve scaling—no communication. Scales well ~L_{max}, the partial wave cutoff.

Angular Quadrature—Gauss-Legendre



1. Surface discontinuity leads to significant numerical error.
2. **Integration must be broken up: $I = \int_{S1} + \int_{S2}$**
3. Gauss-quadrature dominates the run time.
4. Functions are cached for each shell to speed up performance.

Numerical Convergence / Precision

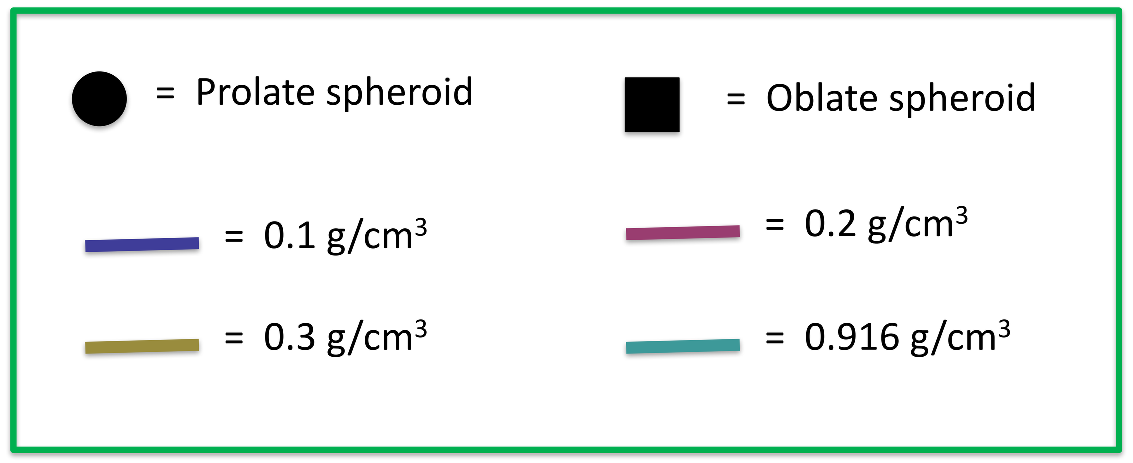


Oblate Spheroid	Extinction efficiency	Scattering Efficiency
IITM+SOV (this)	3.28513 (2)(2)	2.29014 (2)(2)
EBCM [1]	3.2854	2.2903
IITM+SOV [1]	3.2850	2.2901

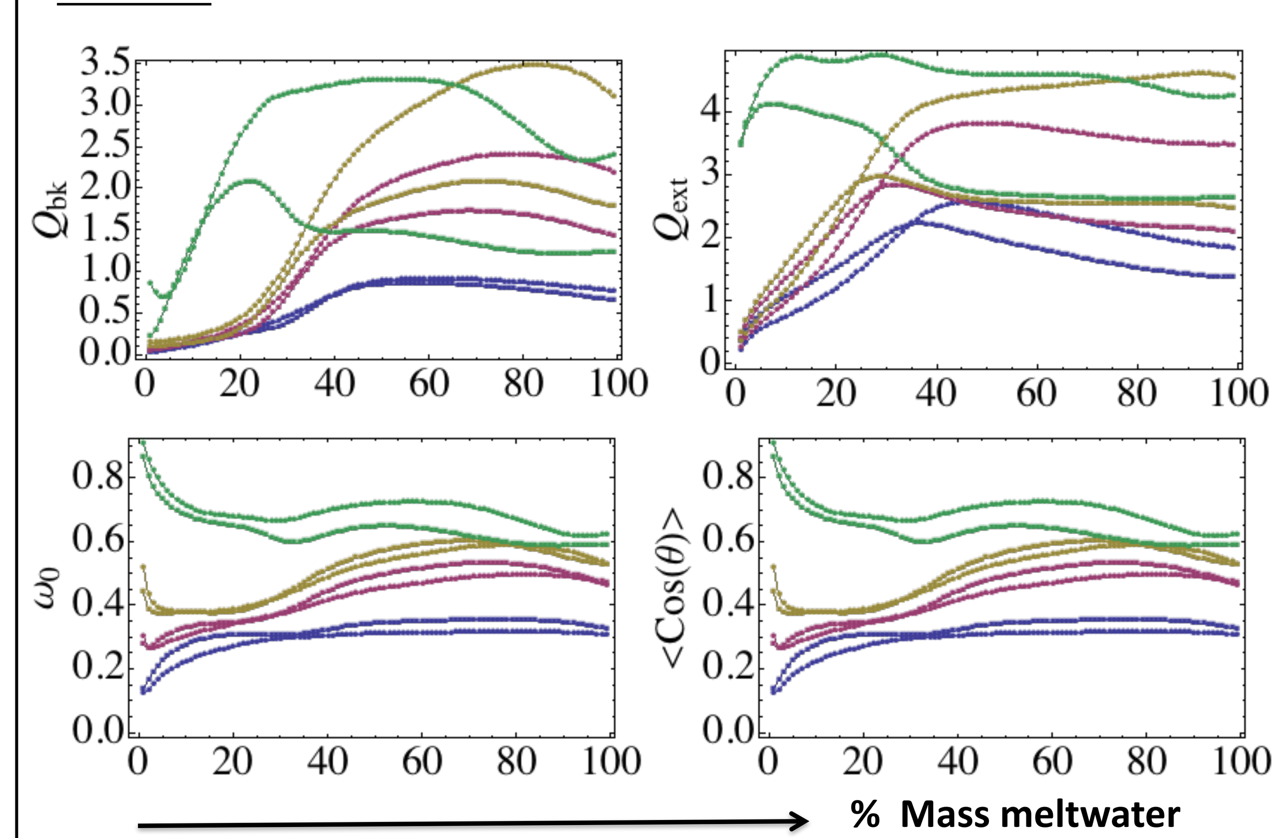
Comparison with results from [1] with error estimates associated with partial wave truncation and the number of radial shells (δl)(δr), respectively.

Mixed-Phase Hydrometeor Spheroids

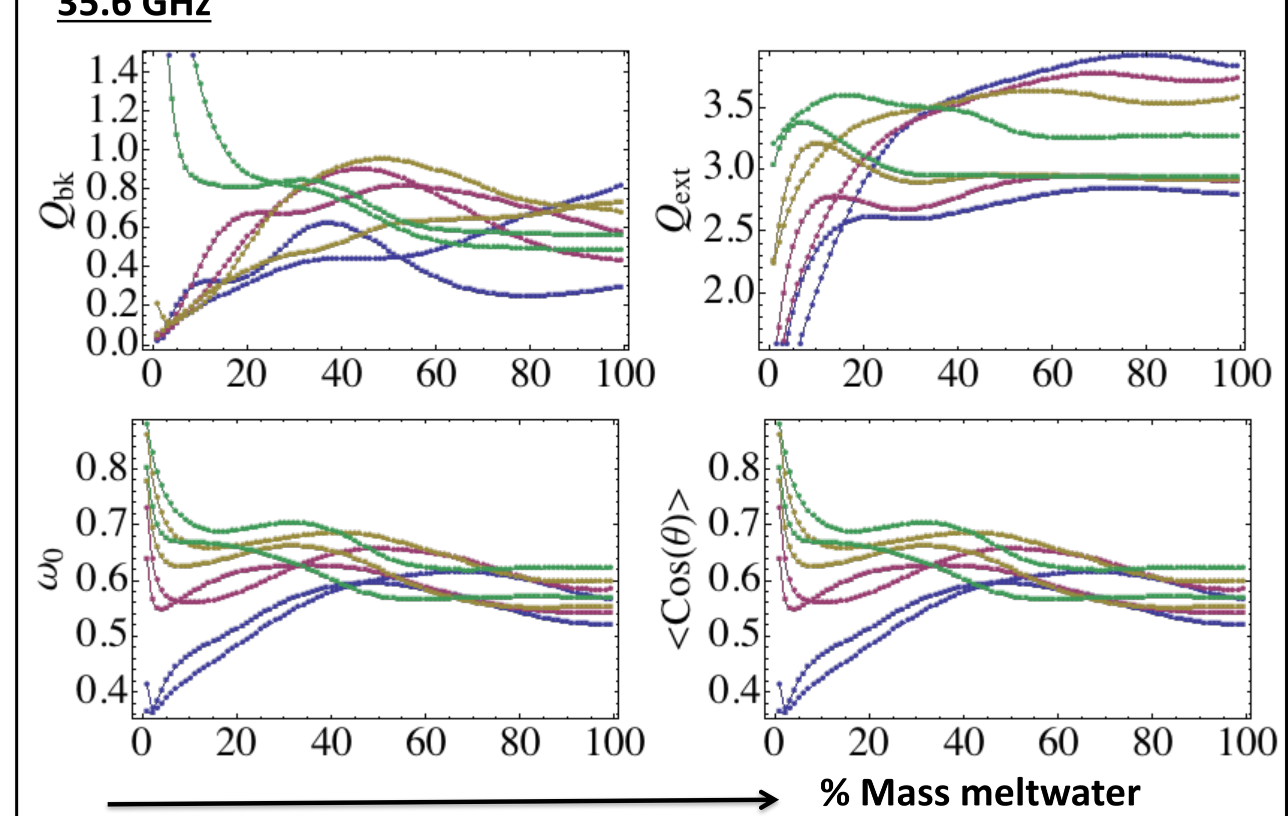
To test and study the behavior of our IITM implementation, we study spheroids with an outer shell of melt water. We consider a solid ice core as well as "fluffy" mixed air-ice cores with densities 0.1, 0.2, and 0.3 g/cm³. Numerical calculations are carried for both prolate and oblate spheroids with a semi-minor to semi-major ratios equal to 1/2(2) for frequencies 13.6 GHz, 35.6 GHz, and 94 GHz. The corresponding size parameters are ~0.25, ~0.64, and 1.7.



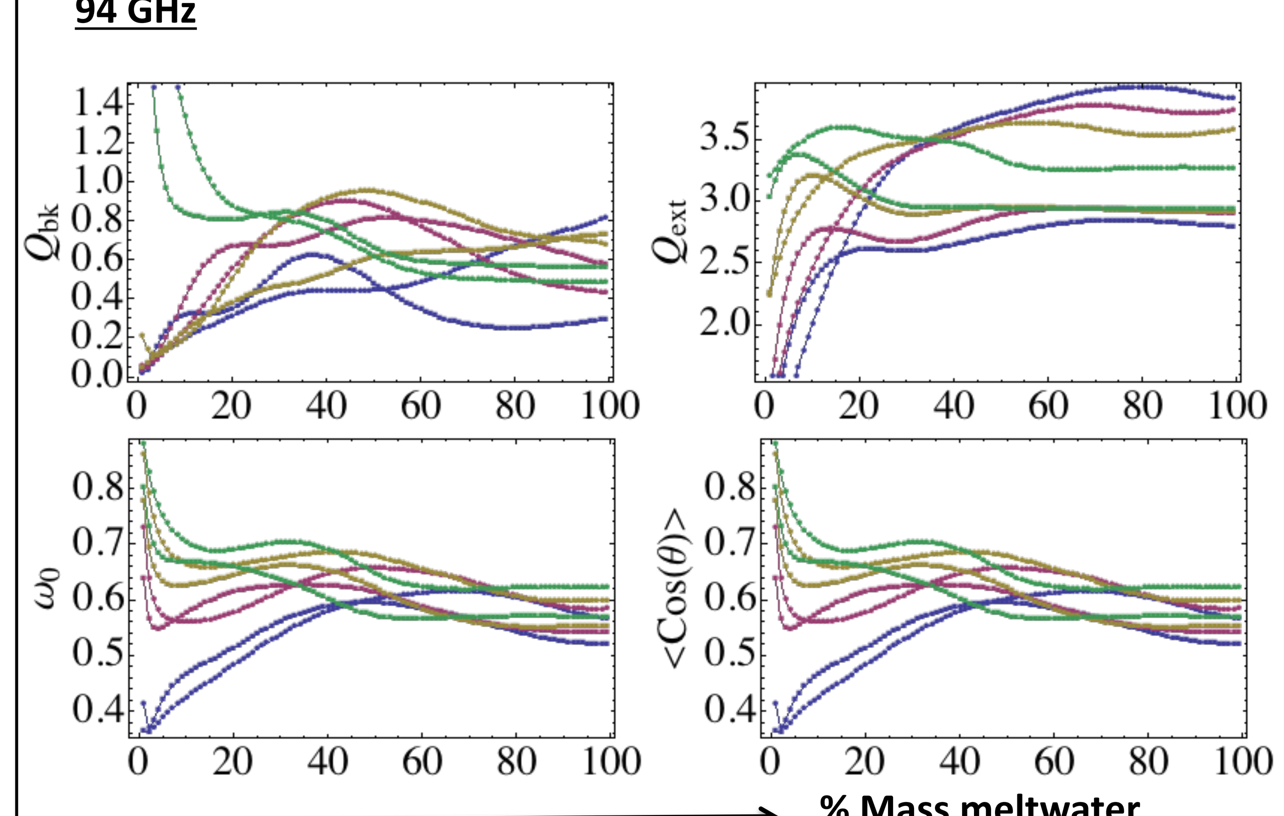
13.6 GHz



35.6 GHz



94 GHz



Irregular Hydrometeors

T-Matrix methods are computationally more efficient than Discrete Dipole Approximations (DDAs) for scattering targets that possess a high degree of symmetry. For the case of spheroids shown, it only takes a few minutes on 32 cores to compute the orientation averaged scattering properties. The reason for this is that the T-Matrix becomes sparse and structured, reducing the problem size significantly. Irregular particles incur a substantially larger cost as the maximum allowed angular momentum, L_{max}, increases, since the block diagonal structure is lost. Specifically, the computational cost for axial and no symmetry:

	Axial symmetry	No symmetry
FLOPS	$\sim L_{max}^4 \times \#\,shells$	$\sim L_{max}^6 \times \#\,shells$

A further complication is that surface integrals carried out over the sphere become highly discontinuous for irregular particles. In this case, the "nice" properties of Gauss-Legendre quadrature are no longer true, and partitioning the integrals would require a numerical routine to find the boundaries.

Conclusions

An efficient C++ parallel IIITM code has been developed. This code accurately computes SSPs of non-uniform hydrometeors with rotational symmetry. The code parallelizes by taking advantage of the block structure induced by rotational symmetry about the z-axis by distributing blocks among MPI processes. Long double precisions was required to achieve satisfactory accuracy. This IITM code will allow the efficient computation of mixed-phase rotationally-symmetric hydrometeors necessary for modeling precipitating systems.

It is challenging to use T-Matrix Methods for irregular hydrometeors, and it is not clear that numerical precision and compute constraints will allow accurate SSPs to be computed using this method. Currently, DDA approaches are employed.

References

[1] Bi, L., P. Yang, G.W. Kattawar, and M.I. Mishchenko, 2013: Efficient implementation of the invariant imbedding T-matrix method and the separation of variables method applied to large nonspherical inhomogeneous particles. *J. Quant. Spectrosc. Radiat. Transfer*, **116**, 169-183, doi:10.1016/j.jqsrt.2012.11.014

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Contact Information

Craig Pelissier, PhD
Science Systems and Applications Incorporation
NASA Goddard Space Flight Center, Code 606, Greenbelt, MD
craig.s.pelissier@nasa.gov