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Motivation

Data assimilation algorithm:

$$x^{a} = x^{f} + K(y - Hx^{f})$$

$$K = PH^T (HPH^T + R)^{-1}$$

- Initialization of P is very important especially for variational methods
- Incorrect specification of forecast errors can lead to a degraded forecast

The questions:

- Does hydrostatic balance holds in forecast perturbations at high resolutions?
- If it does then to what extent?

Case study

- Formulation

Case study of 3h window on 27/07/2008 over southern UK:

- Model used is Met Office Global and Regional Ensemble Prediction System (MOGREPS)
- Model has 1.5km horizontal resolution
- MOGREPS uses ETKF methodology to determine a set of perturbed forecasts and 1.5km UM forward model
- Model is initialized with a reconfigured ensemble from 24km to 1.5km at 18Z
- ▶ 24 ensemble member forecasts available at 19Z, 20Z, 21Z
- Domain size $360 \times 288 \times 70$

- Case study
- Data selection



Available fields for all vertical levels are

- Exner pressure Π
- ▶ potential temperature θ
- and specific humidity q

Case study

Ensemble set-up

Let the vertical state vector be defined as

$$\mathbf{x} = (\Pi_{0,\dots,k-1}, \theta_{0,\dots,k-1}, q_{0,\dots,k-1}),$$

where k = 70 is number of vertical levels and an ensemble is defined as

$$\mathbf{X} = [\mathbf{x}_0, \mathbf{x}_1, ..., \mathbf{x}_{N-1}] \in \mathcal{R}^{3k imes N}$$

where N = 24 is the number of ensemble members.

The ensemble mean is defined as $\overline{\mathbf{x}}$. Thus, the ensemble perturbations are given by

$$\mathbf{X}'_i = \mathbf{X}_i - \overline{\mathbf{x}}$$

where i = 0, ..., N - 1.

Case study

-Derivation of hydrostatic balance in perturbations

Hydrostatic balance is given by (e.g. see Wallace 2006),

$$\frac{dp}{dz} = -\frac{gp}{RT}$$

where p(z) is pressure, T(z) is temperature, z is vertical height, and $R = 287.06 \text{ J K}^{-1}\text{kg}^{-1}$ is the gas constant for dry air. Using expression for Exner pressure $\Pi = \left(\frac{p}{p_0}\right)^{R/c_p} = \frac{T}{\theta^{\nu}}$ and virtual potential temperature $\theta^{\nu} = \theta \left(1 + (\epsilon^{-1} - 1)q\right)$, hydrostatic equation above can be written as follows

$$\frac{d\Pi}{dz} = -\frac{g}{c_p} \left(1 + (\epsilon^{-1} - 1)q\right)^{-1} \theta^{-1}.$$

Case study

- Derivation of hydrostatic balance in perturbations

By decomposing $\Pi = \overline{\Pi} + \Pi'$, $\theta = \overline{\theta} + \theta'$ and $q = \overline{q} + q'$ above equation may be linearized giving a first order approximation to hydrostatic equation for the perturbations:

$$\frac{d\Pi'}{dz} = \frac{\mathsf{g}}{\mathsf{c}_{\mathsf{p}}} \left[\frac{(\epsilon^{-1} - 1)q'}{(1 + (\epsilon^{-1} - 1)\bar{q})^2 \bar{\theta}} + \frac{\theta'}{(1 + (\epsilon^{-1} - 1)\bar{q})\bar{\theta}^2} \right]$$

Rearranging gives

$$heta_{\mathcal{H}}' = -rac{(\epsilon^{-1}-1)q'\overline{ heta}}{1+(\epsilon^{-1}-1)\overline{q}} + rac{c_{p}}{g}rac{d\Pi'}{dz}\overline{ heta}^{2}(1+(\epsilon^{-1}-1)\overline{q}).$$

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Perturbation correlation



- Compute θ'_H for each column
- Compare correlation matrices of θ' and θ'_H
- Examine explained variances and error flow
- Aggregate data around Conv19Z and NonConv points to examine hydrostatic balance as a function of horizontal scale

- Results

Perturbation correlation

Correlation matrices for 19Z at 1.5km resolution



(a) θ' , Non-Conv (b) θ'_H , Non-Conv



- Results

Perturbation correlation

Correlation matrices for 19Z at 4km and 12km resolution



(c) θ' , Conv12km (d) θ'_H , Conv12km

- Results

- Explained variance

Balance measure

The explained variance is given by

$$E(z) = \left(1 - \frac{\sigma_U^2(z)}{\sigma^2(z)}\right)$$

where σ^2 is the grid-point variance of θ' and σ_U^2 is the variance of the unbalanced part of the perturbations, i.e. $\theta'_U = \theta' - \theta'_H$, and z is the vertical level. Thus, if $E \approx 1$ then perturbations are close to hydrostatic balance and if $E \approx 0$ then they are imbalanced.

- Results

- Explained variance

Explained variance at 1.5km resolution



- Results

- Explained variance

Explained variance at coarser resolution



- Results

Relative RMS error

$$rel.error = rac{\sqrt{(heta'_H - heta')^2}}{| heta'_H|} imes 100$$



- At 1.5km resolution hydrostatic balance does not hold in the perturbations in the regions of convection but it does hold in the regions where convection is not present.
- This suggests that hydrostatic balance should be relaxed around convective columns in the correlation matrices at 1.5km resolution. A way of achieving this would be by redesigning the control variable transform in UM.
- ➤ ≈ 20km horizontal resolution is the limit at which the hydrostatic balance becomes valid over the entire domain.

Future work

- Investigate balance properties in P using an idealised 1+2D convective model and EnSRF
- Further, how balances are affected by applying localization
- Test performance of EnSRF when applied to a convective model

Thank You!



Fig. 7: Setup and flow of the MOGREPS ensemble prediction system. Here t₀ = 18Z, t₁ = 19Z, t₂ = 20Z, and t₃ = 21Z all on 27/07/2008 available at (**)





(c) 21Z