Towards quantifying variance captured by observations in the California Current System

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Many thanks:
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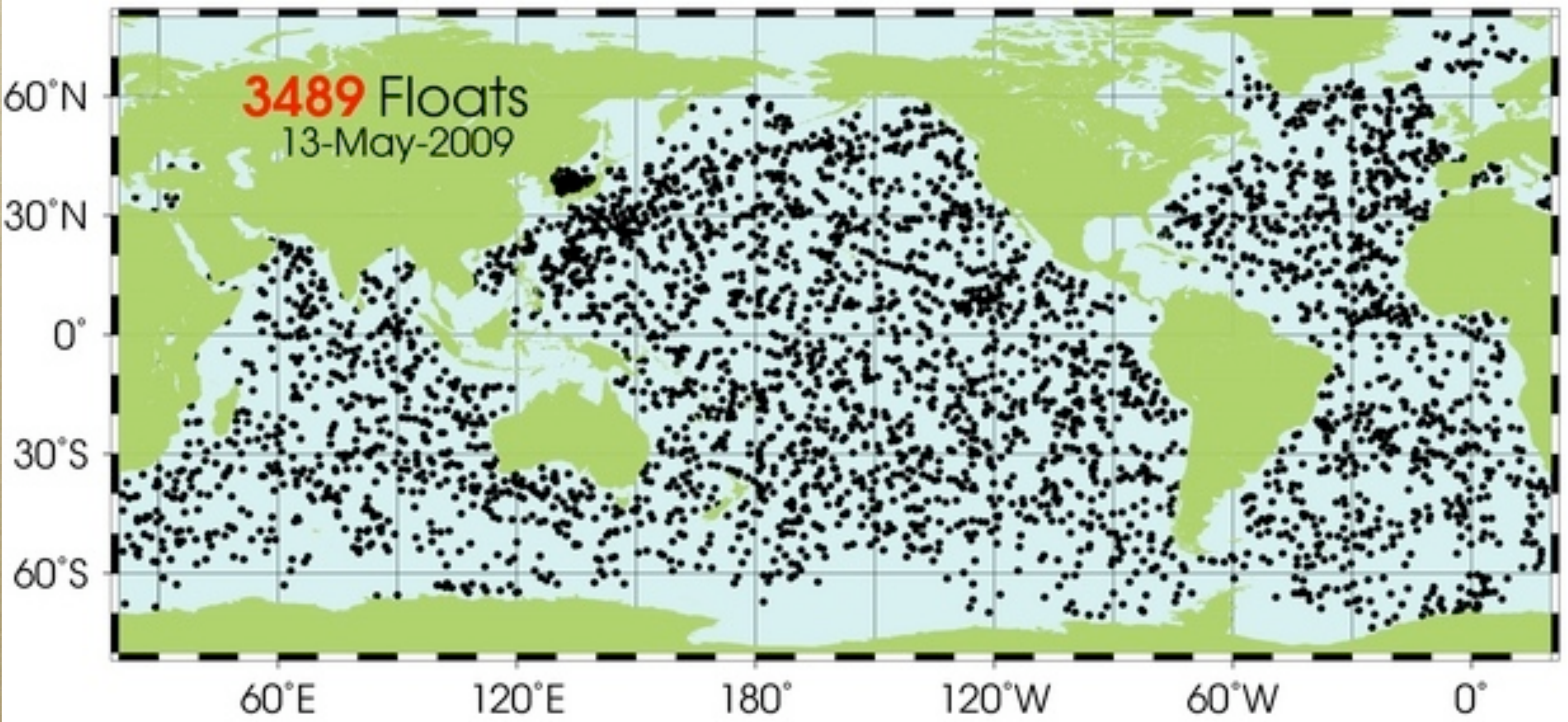

3489 Floats
13-May-2009
60°N
30°N
0°
30°S
60°S
60°E
120°E
180°
120°W
60°W
0°

UNIVERSITY OF HAWAIʻI
MĀLAMALAMA
1907
UA MAU KE EA O KA ʻĀINA I KA PONO

Model/Obs Comparison

Innovation Vector: $\mathbf{d} = \mathbf{y}^o - \mathcal{H}\mathbf{x}^b$,

where, $\mathbf{y}^o$ are observations, $\mathbf{x}^b$ is background circulation, $\mathbf{x}^t$ truth

Given errors of:

$$\epsilon^b = \mathbf{x}^b(0) - \mathbf{x}^t(0)$$
$$\epsilon^o = \mathbf{y}^o - \mathcal{H}\mathbf{x}^t$$

Then,

$$\left\langle \mathbf{d}\mathbf{d}^T \right\rangle = \left\langle \left(\epsilon^o - \mathbf{G}\epsilon^b\right)\left(\epsilon^o - \mathbf{G}\epsilon^b\right)^T \right\rangle$$
$$= \mathbf{R} + \mathbf{G}\mathbf{D}\mathbf{G}^T$$

where $\mathbf{G} = \mathbf{H}\mathbf{M}$

Representers & Array Modes

Bennett (2001) introduces the representer functions, which are:

$$\mathbf{H}\mathcal{R} = \mathbf{G}\mathbf{D}\mathbf{G}^T$$

where,

$$\mathcal{R} = (\mathbf{r}_m)$$

hence,

$$\mathbf{r}_m = \mathbf{M}\mathbf{D}\mathbf{G}^T \delta(\mathbf{y}^o_m)$$

Decompose the stabilized representer matrix:

$$\mathbf{R} + \mathbf{G}\mathbf{D}\mathbf{G}^T = \mathbf{Z}\mathbf{\Phi}\mathbf{Z}^T$$

$$\Psi_k = \sum_{j=1}^{M} \mathbf{Z}_{jk}\mathbf{r}_j$$

Procedure

- Generate: $\mathbf{Z}\mathbf{\Phi}\mathbf{Z}^T$ via Lanczos
- Inspect $\mathbf{\Phi}$ for K dominant modes
- Identify M dominant elements of $\mathbf{Z}_k$
- Generate: $\mathbf{r}_m$ for $m = (1, \ldots, M)$
- Construct: $\mathbf{\Psi}_k$ for $k = (1, \ldots, K)$

Assimilation

Traditional: solve for an analysis estimate via:

$$\left(\mathbf{R} + \mathbf{G}\mathbf{D}\mathbf{G}^T\right)\beta = \mathbf{d}$$

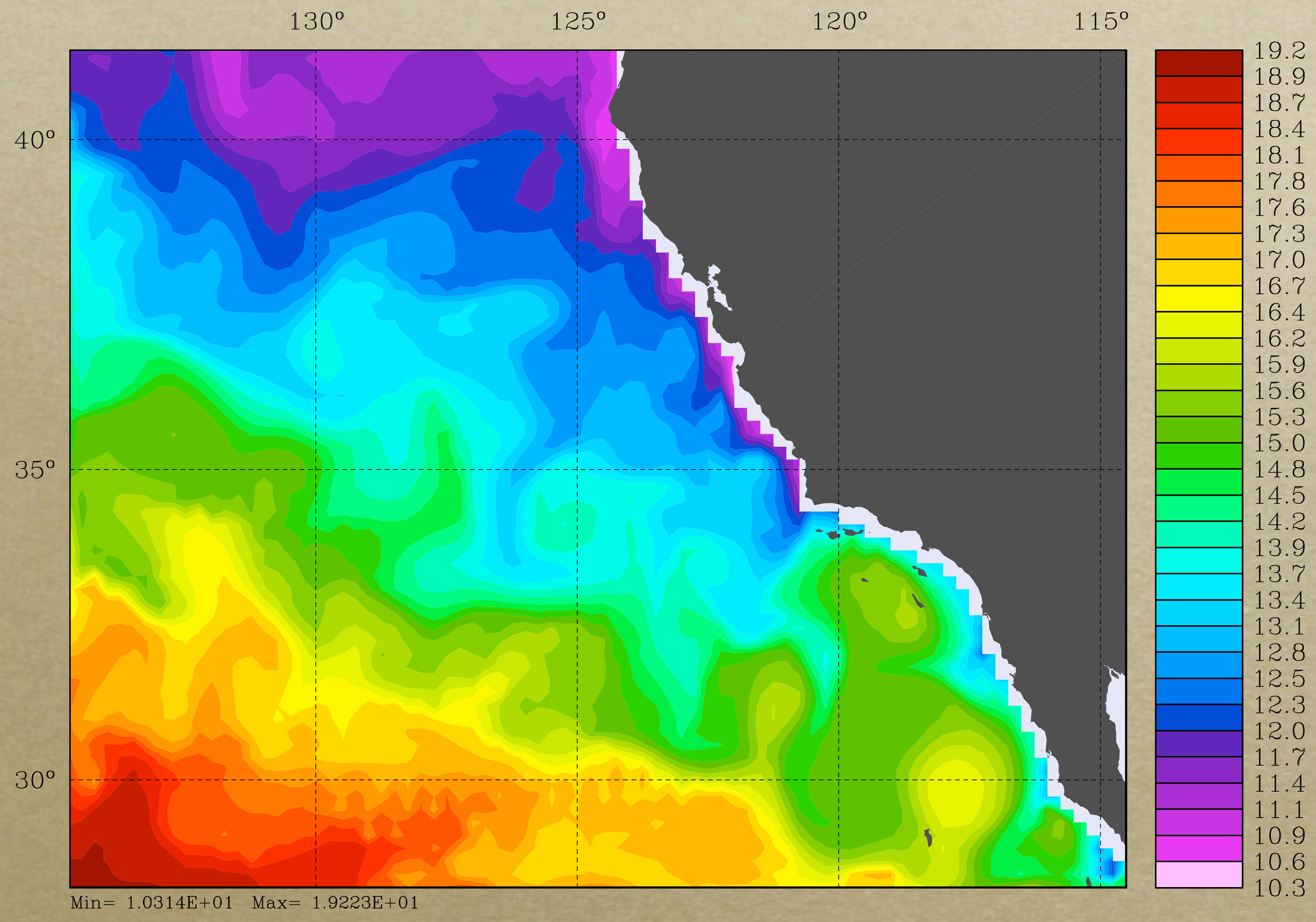
and apply,

$$\mathbf{x}^a = \mathbf{x}^b + \mathcal{R}\beta$$

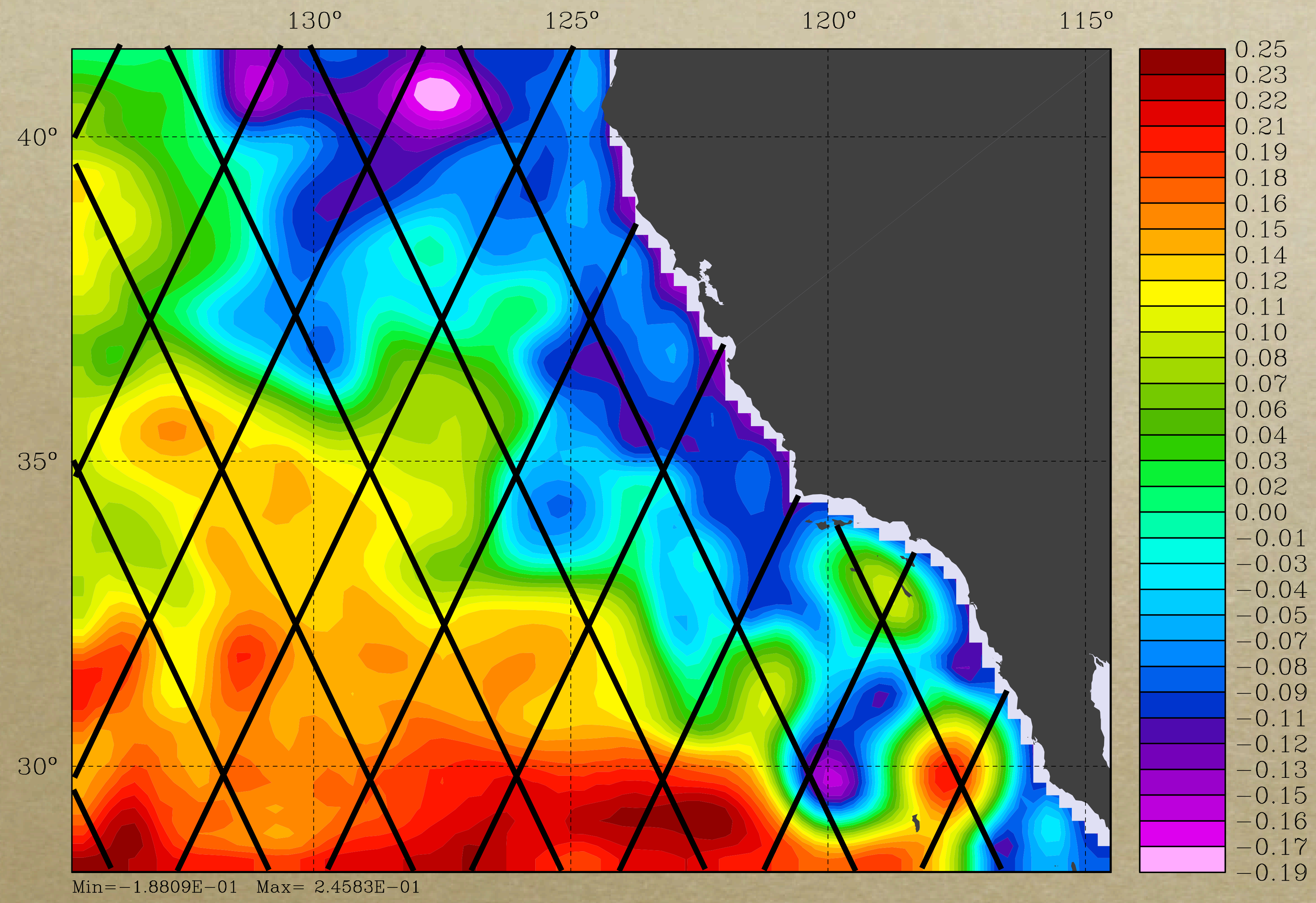
This is equivalent to the combination of array modes (Bennett, 1985):

$$\mathbf{x}^a = \mathbf{x}^b + \sum_{k=1}^{K} \lambda_k^{-1} \mathbf{Z}_k^T \mathbf{d} \Psi_k$$

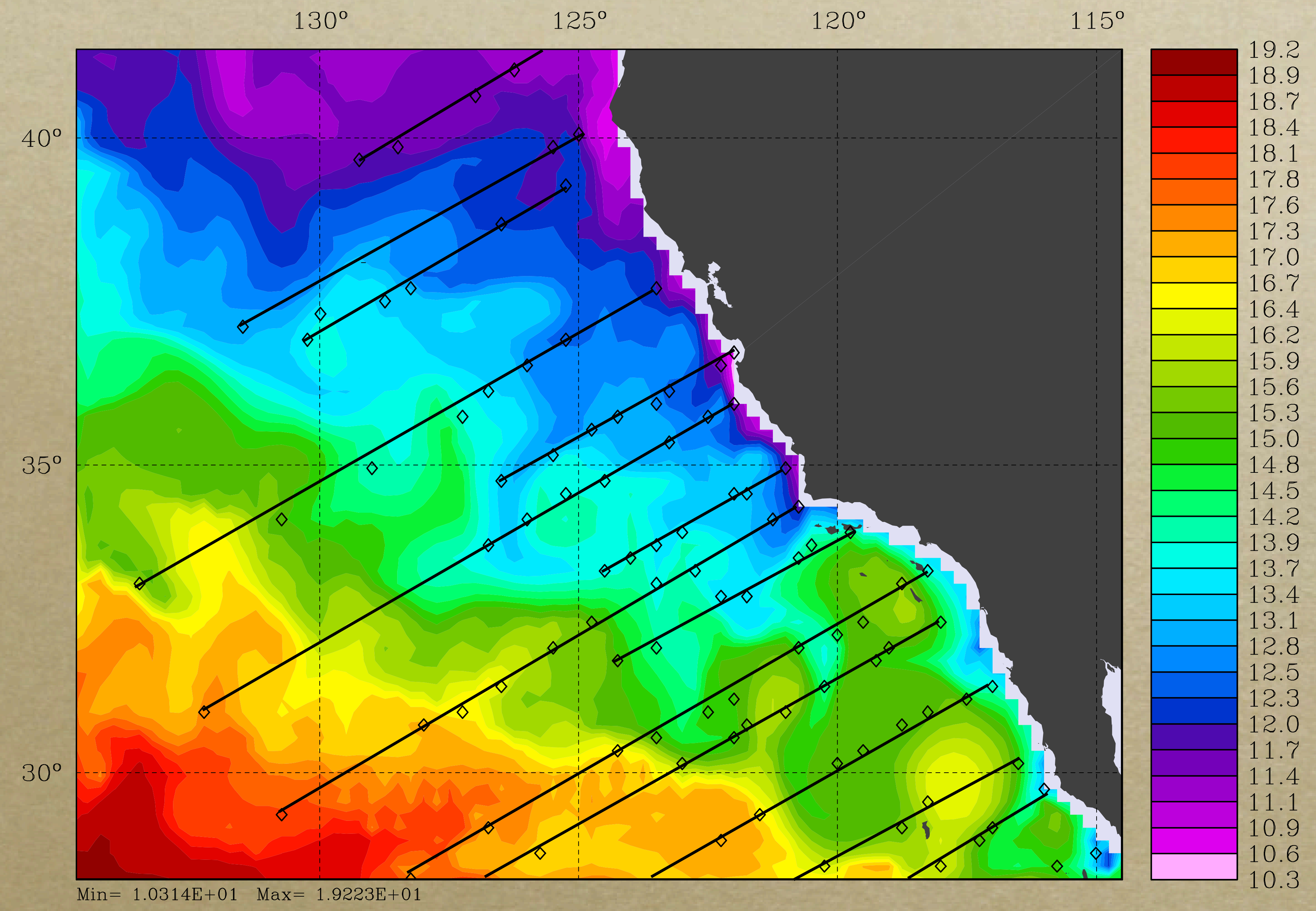
The California Current System



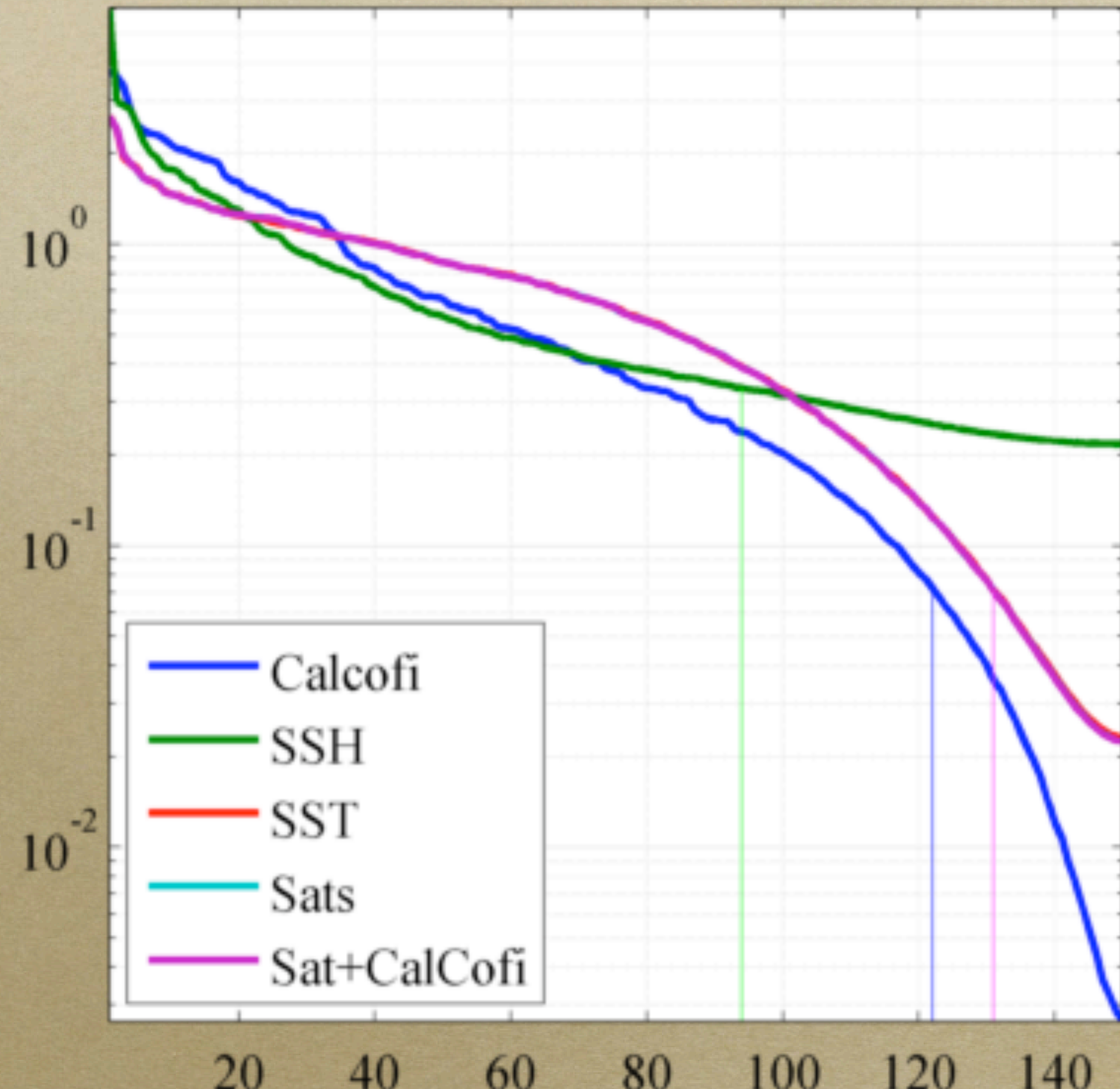
Jason-1


130°
125°
120°
115°
40°
35°
30°
Min=-1.8809E-01 Max= 2.4583E-01
0.25
0.23
0.22
0.21
0.19
0.18
0.16
0.15
0.14
0.12
0.11
0.10
0.08
0.07
0.06
0.04
0.03
0.02
0.00
−0.01
−0.03
−0.04
−0.05
−0.07
−0.08
−0.09
−0.11
−0.12
−0.13
−0.15
−0.16
−0.17
−0.19

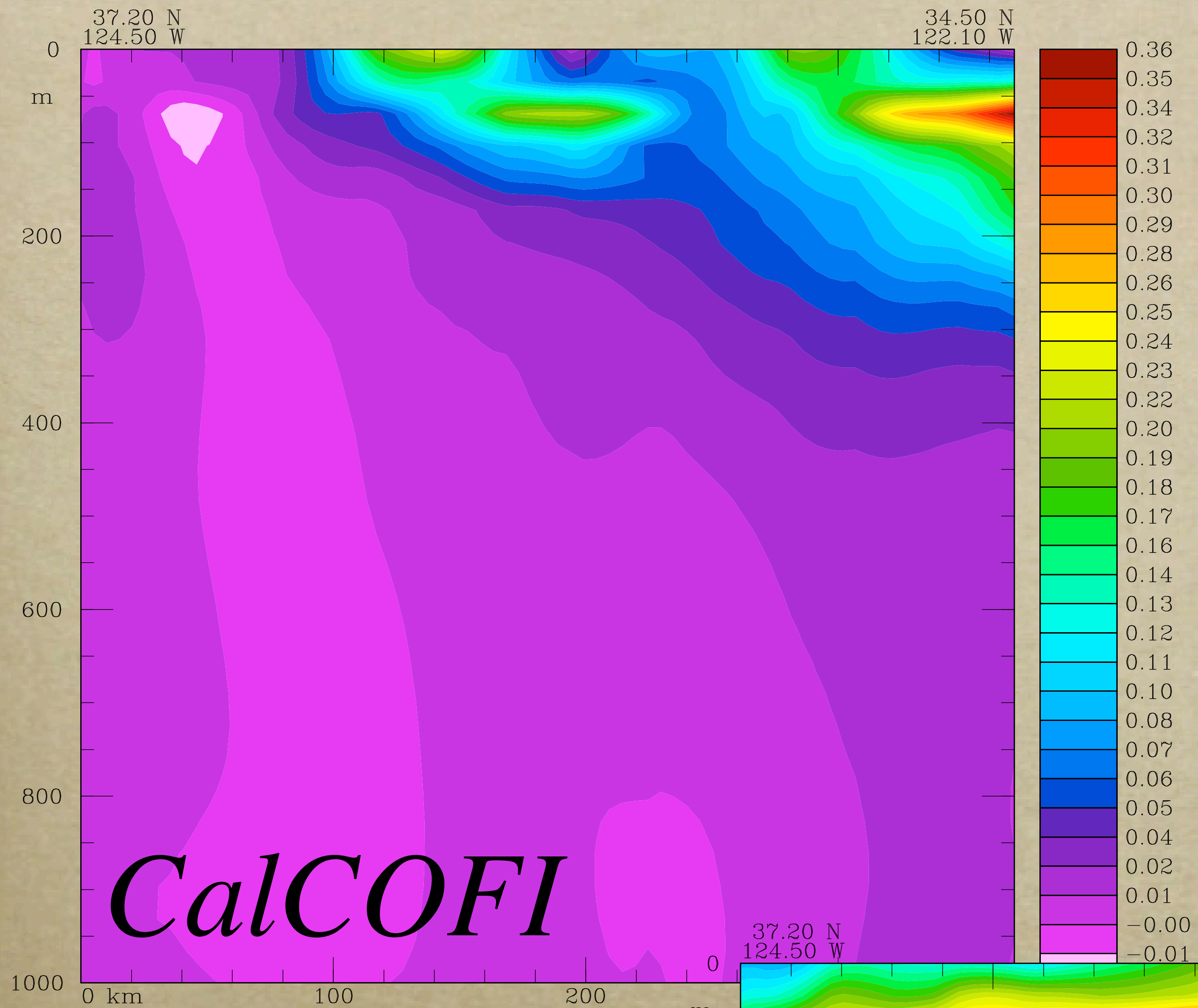
CalCOFI

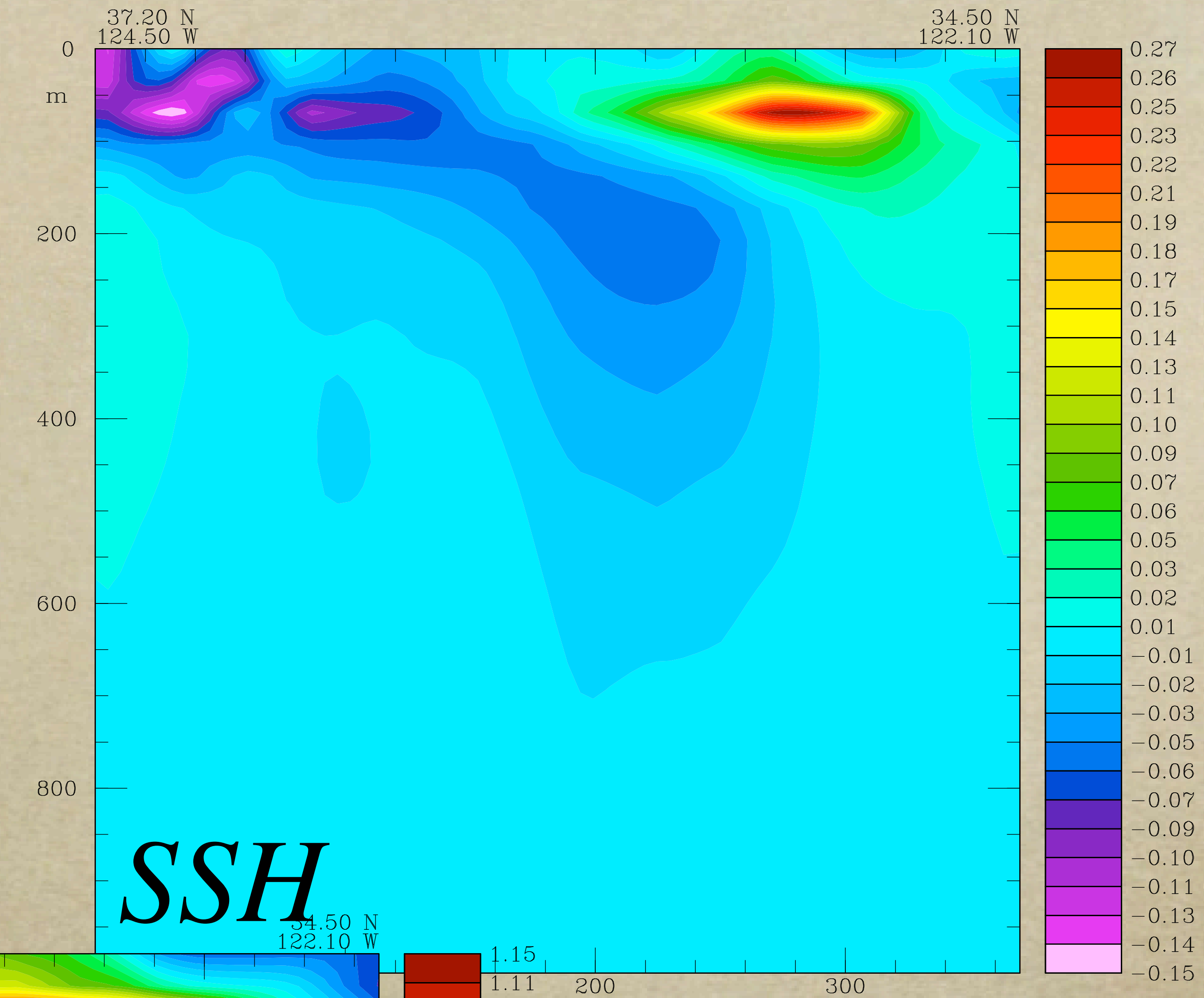


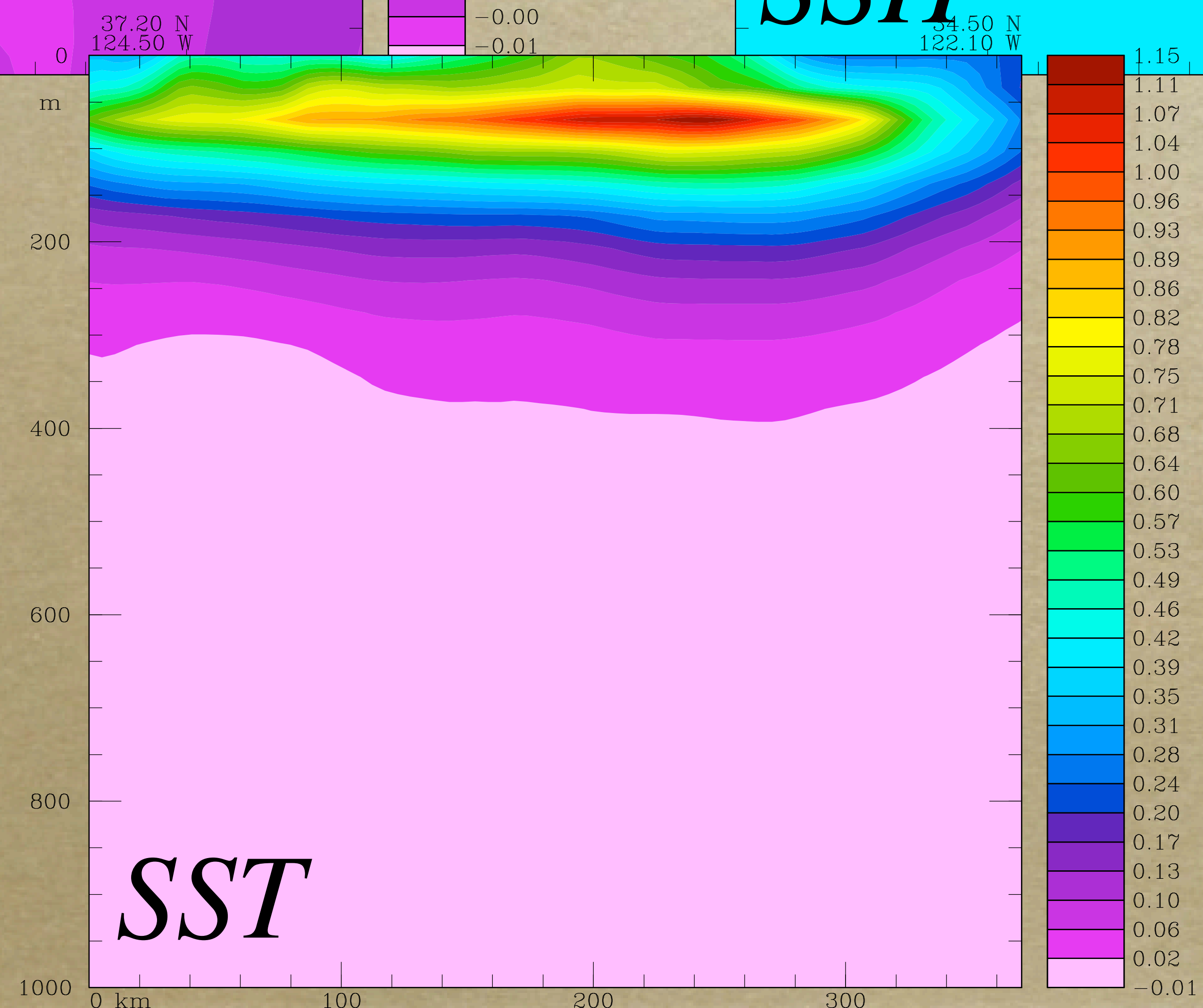
130°
125°
120°
115°
40°
35°
30°
19.2
18.9
18.7
18.4
18.1
17.8
17.6
17.3
17.0
16.7
16.4
16.2
15.9
15.6
15.3
15.0
14.8
14.5
14.2
13.9
13.7
13.4
13.1
12.8
12.5
12.3
12.0
11.7
11.4
11.1
10.9
10.6
10.3
Min= 1.0314E+01 Max= 1.9223E+01



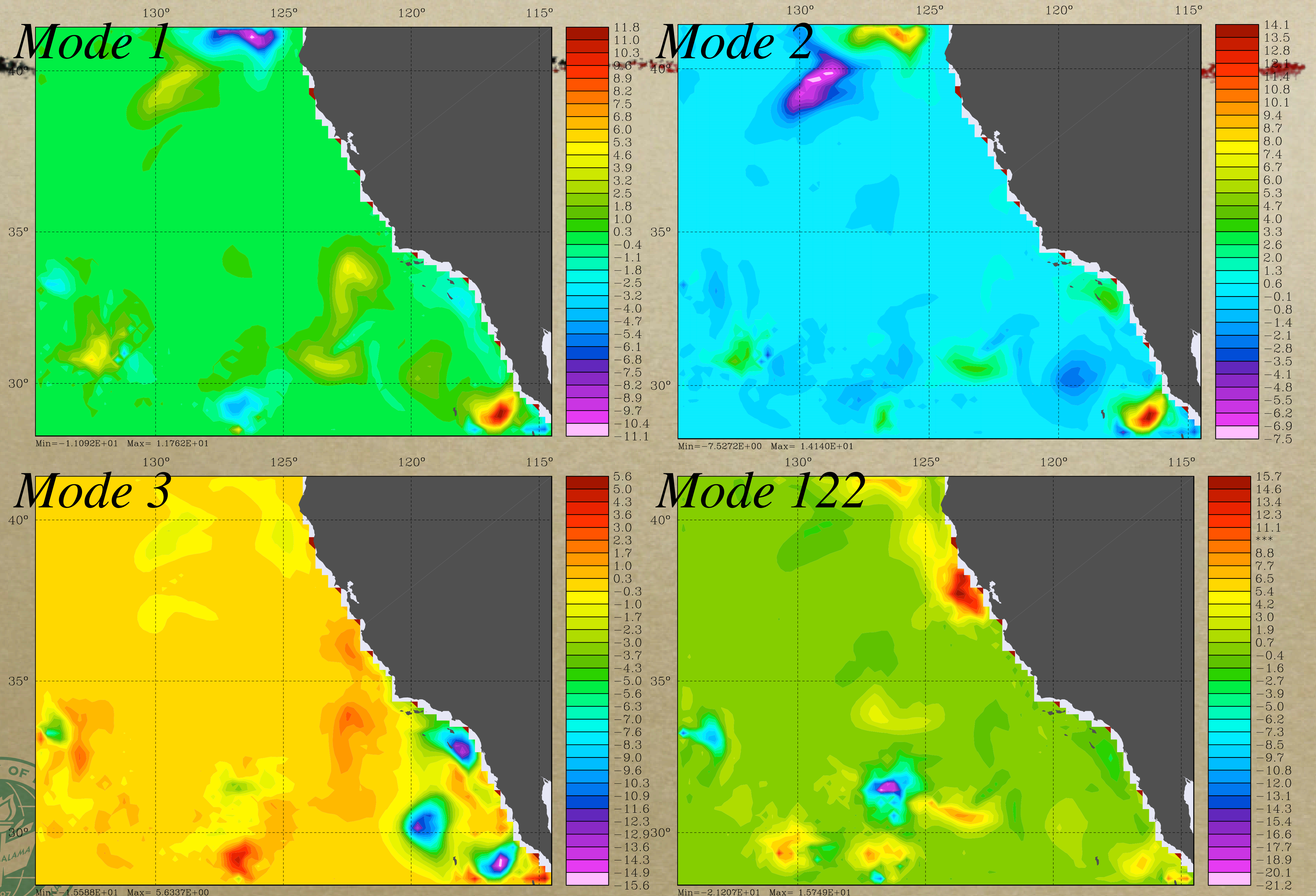
10^0
10^{-1}
10^{-2}
20
40
60
80
100
120
140
Calcofi
SSH
SST
Sats
Sat+CalCofi


CalCOFI
37.20 N
124.50 W
34.50 N
122.10 W
0
m
200
400
600
800
1000
0 km
100
200
0.36
0.35
0.34
0.32
0.31
0.30
0.29
0.28
0.26
0.25
0.24
0.23
0.22
0.20
0.19
0.18
0.17
0.16
0.14
0.13
0.12
0.11
0.10
0.08
0.07
0.06
0.05
0.04
0.02
0.01
−0.00
−0.01

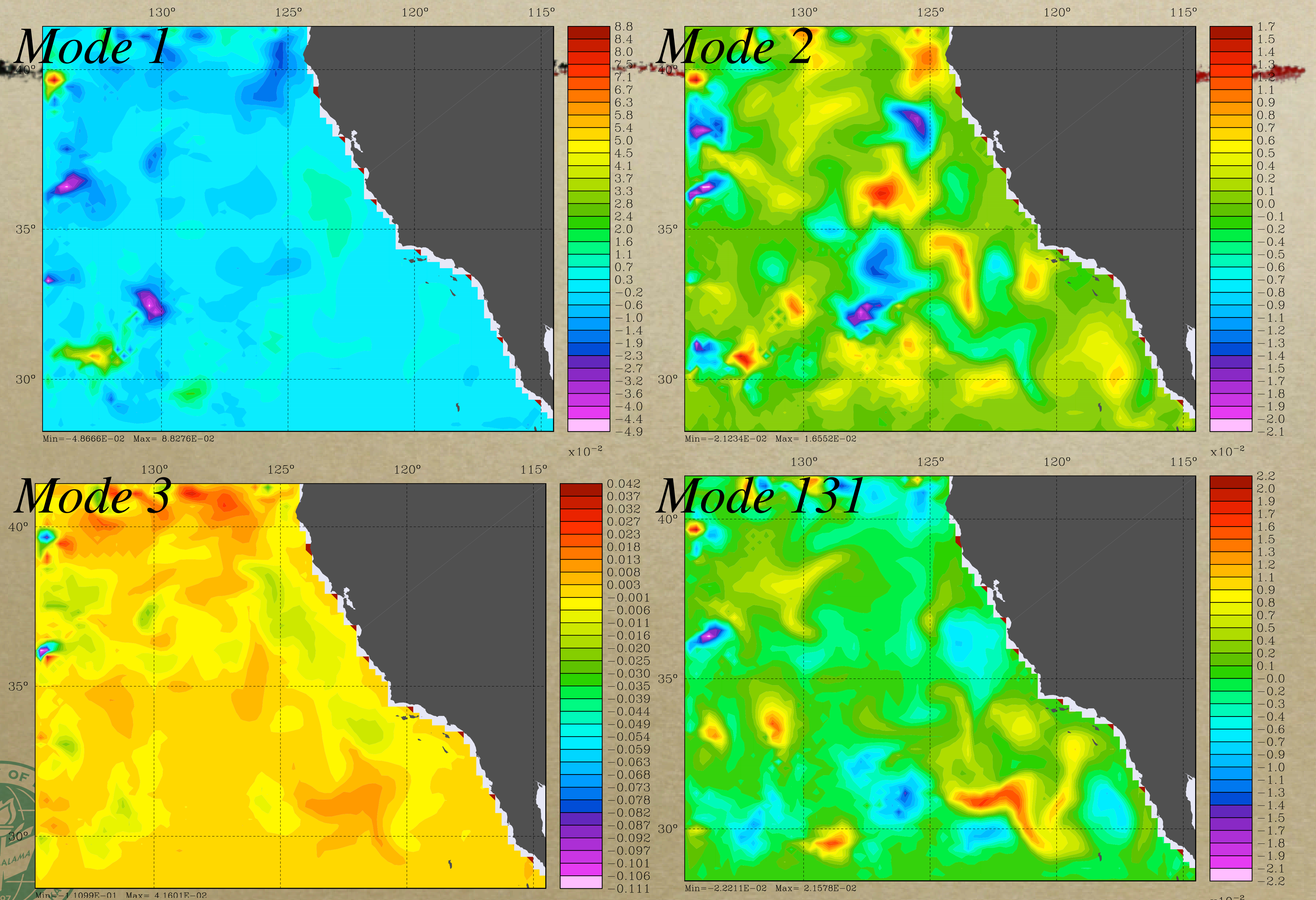

SSH
37.20 N
124.50 W
34.50 N
122.10 W
0
m
200
400
600
800
200
300
0.27
0.26
0.25
0.23
0.22
0.21
0.19
0.18
0.17
0.15
0.14
0.13
0.11
0.10
0.09
0.07
0.06
0.05
0.03
0.02
0.01
−0.01
−0.02
−0.03
−0.05
−0.06
−0.07
−0.09
−0.10
−0.11
−0.13
−0.14
−0.15


SST
37.20 N
124.50 W
34.50 N
122.10 W
0
m
200
400
600
800
1000
0 km
100
200
300
1.15
1.11
1.07
1.04
1.00
0.96
0.93
0.89
0.86
0.82
0.78
0.75
0.71
0.68
0.64
0.60
0.57
0.53
0.49
0.46
0.42
0.39
0.35
0.31
0.28
0.24
0.20
0.17
0.13
0.10
0.06
0.02
−0.01

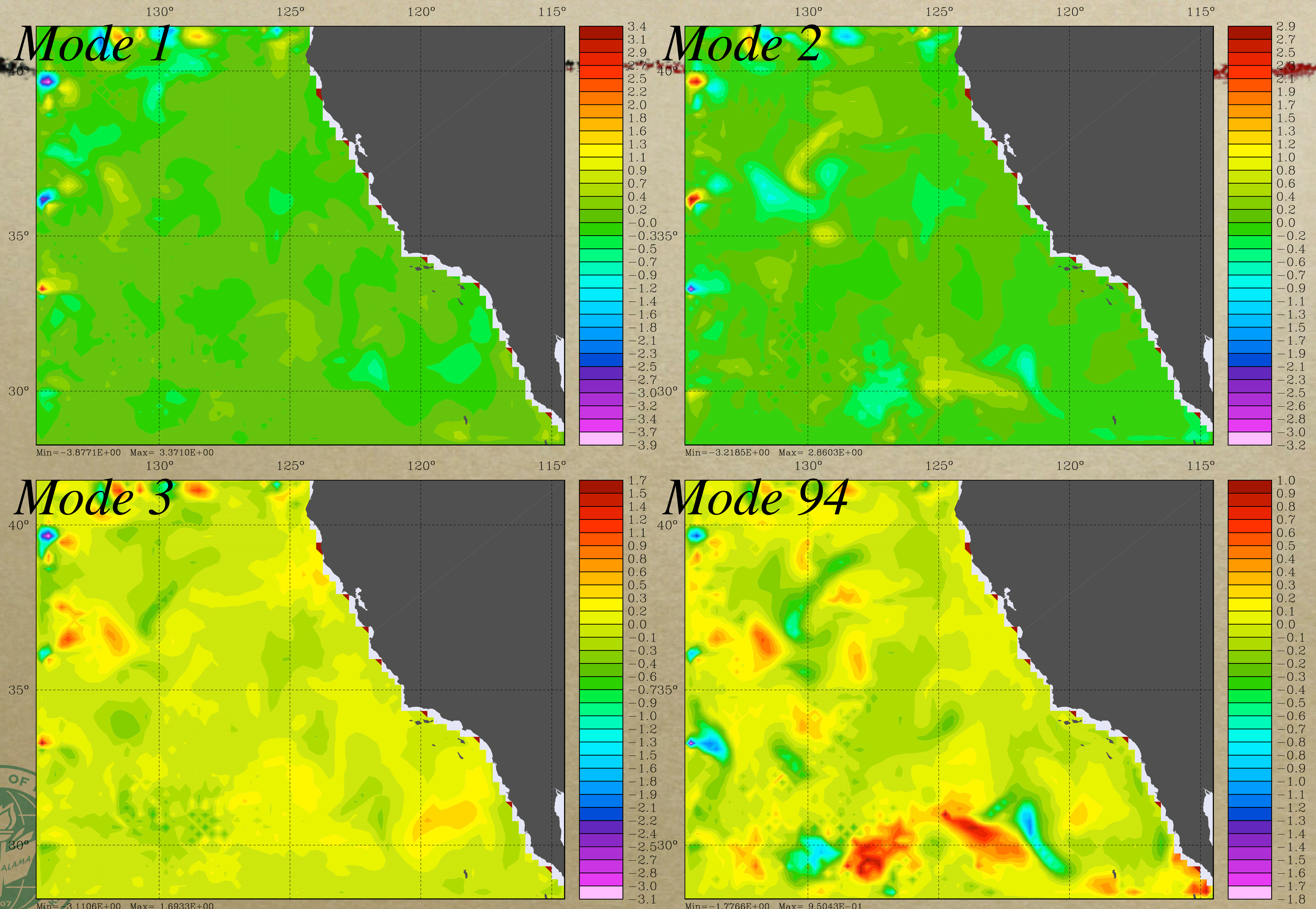
SST Modes from CalCOFI



SST Modes from SST

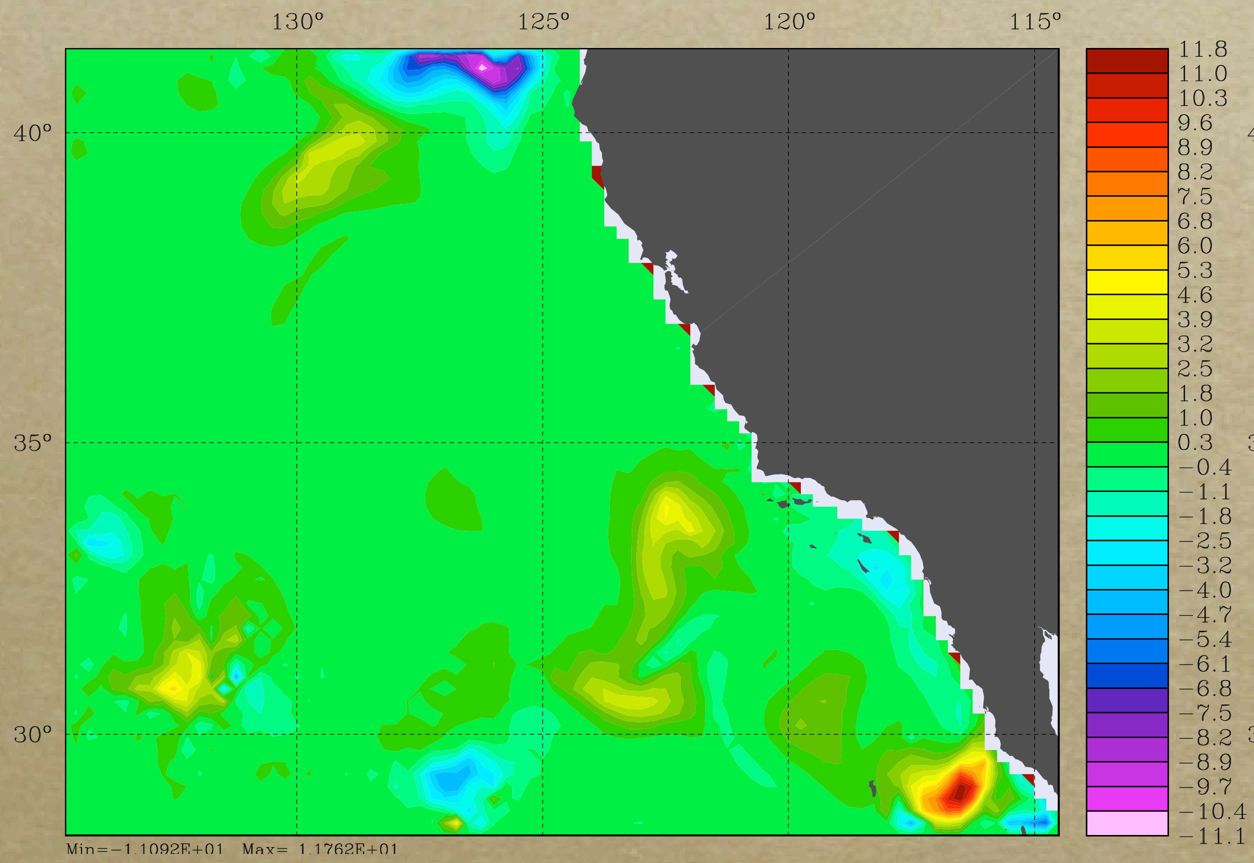


SST Modes from SSH

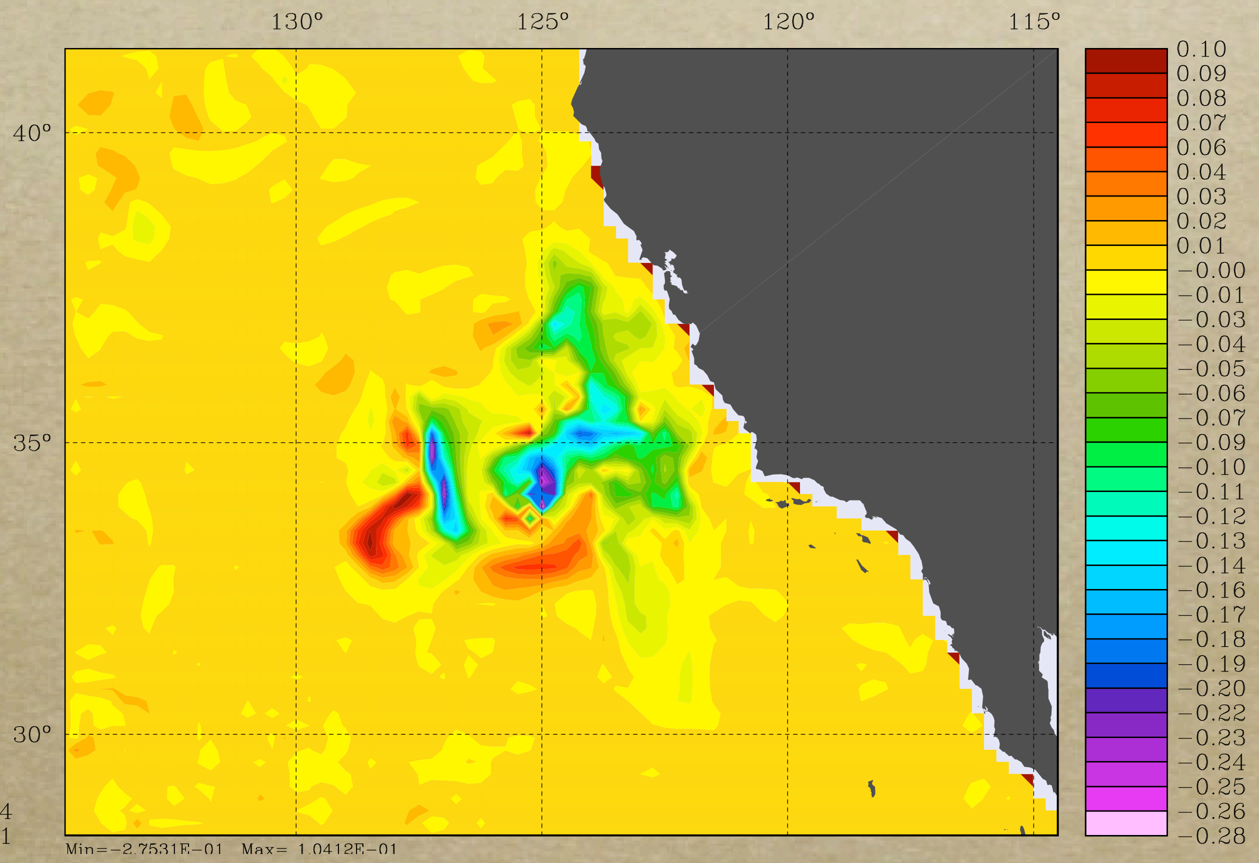


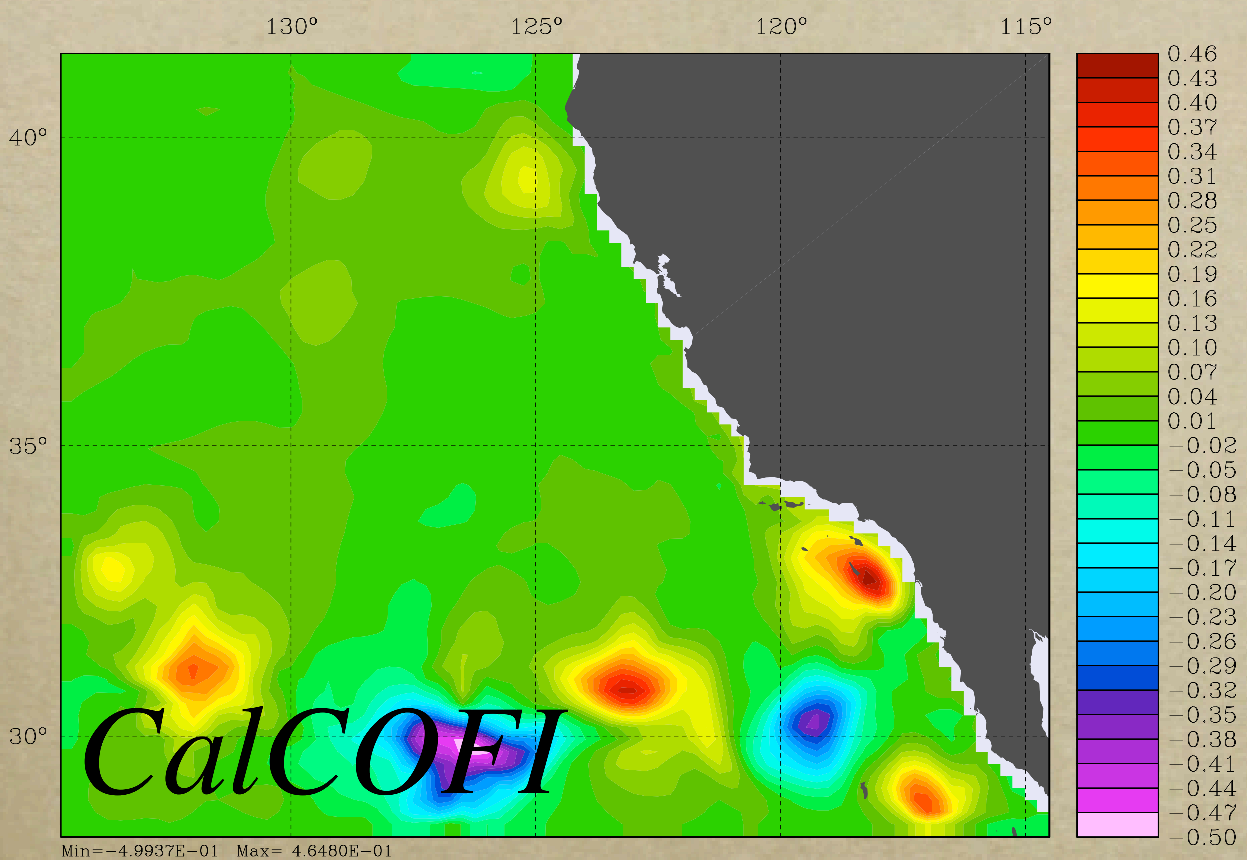
Revised Case

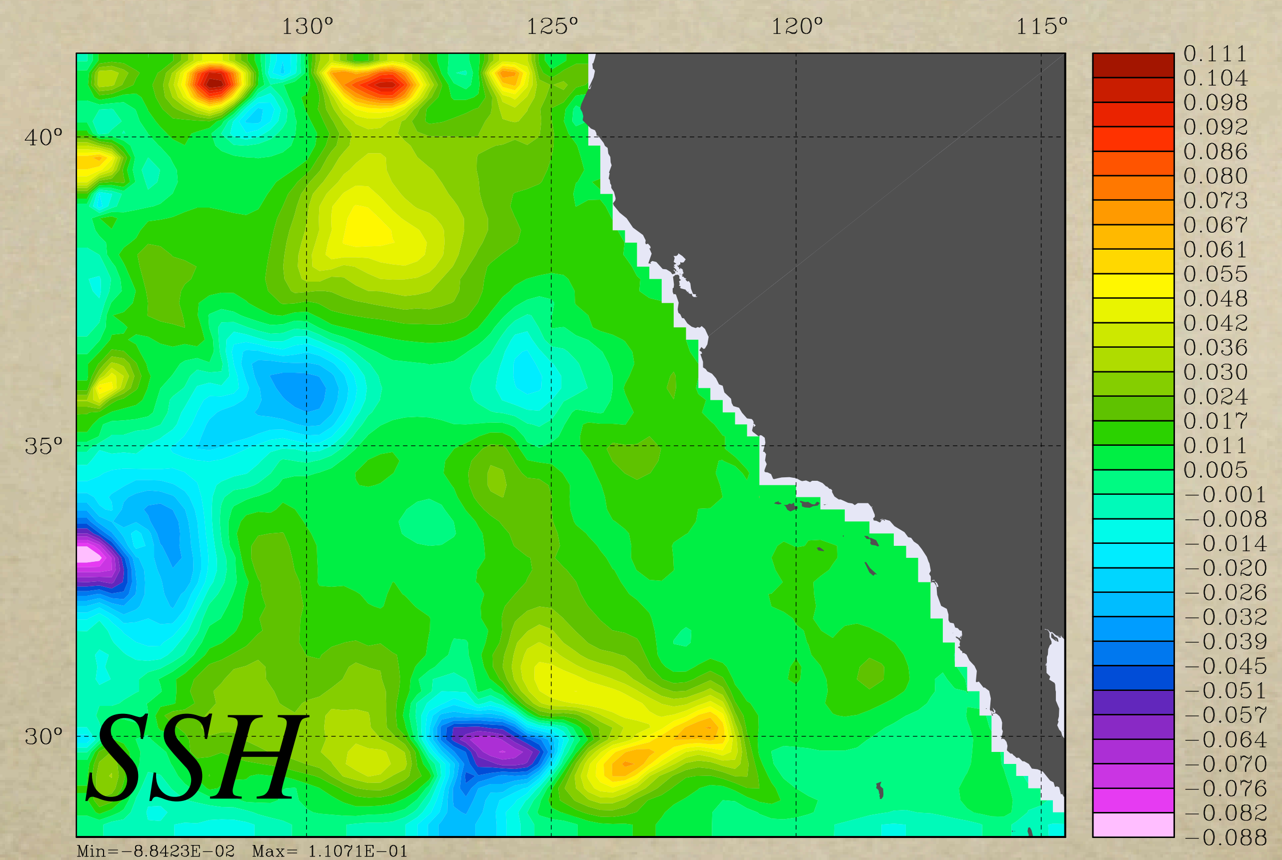
Original

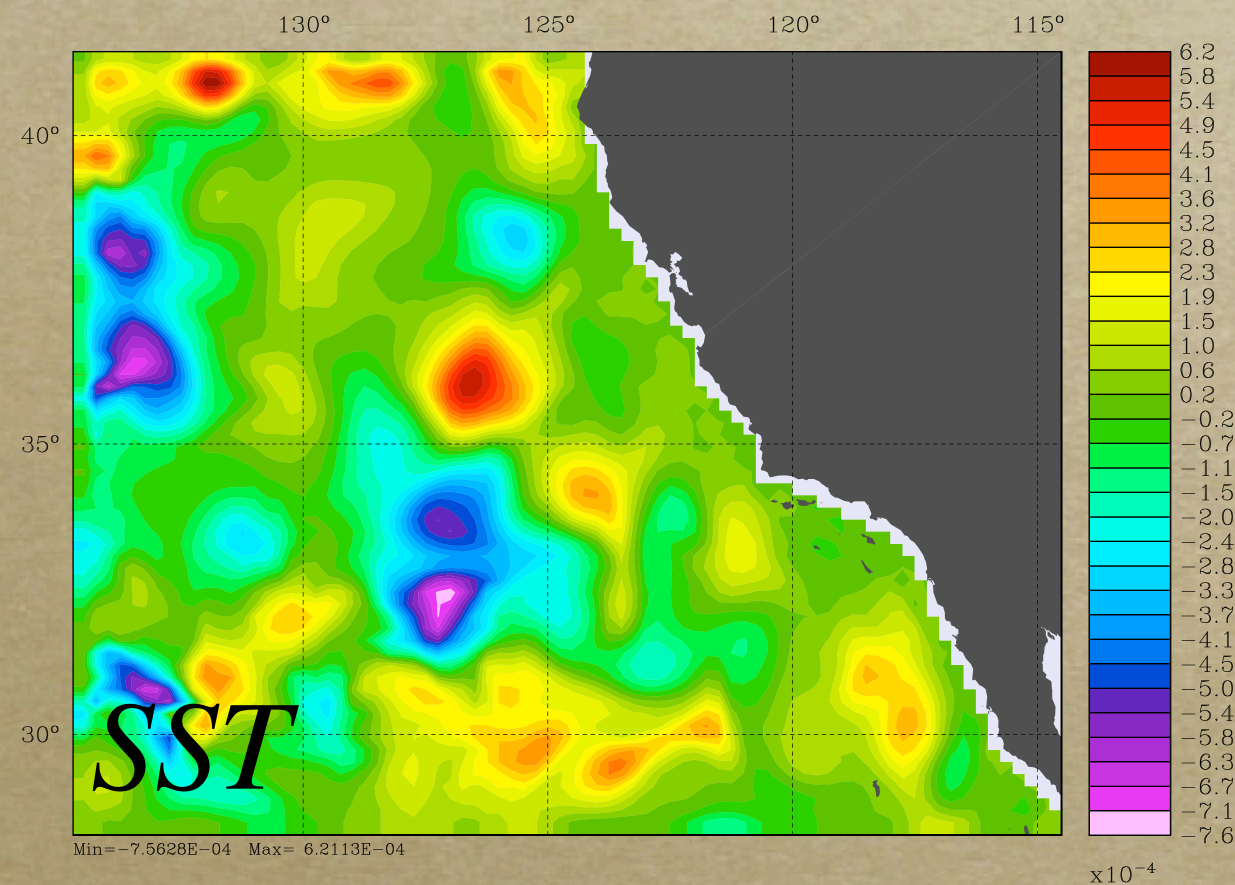


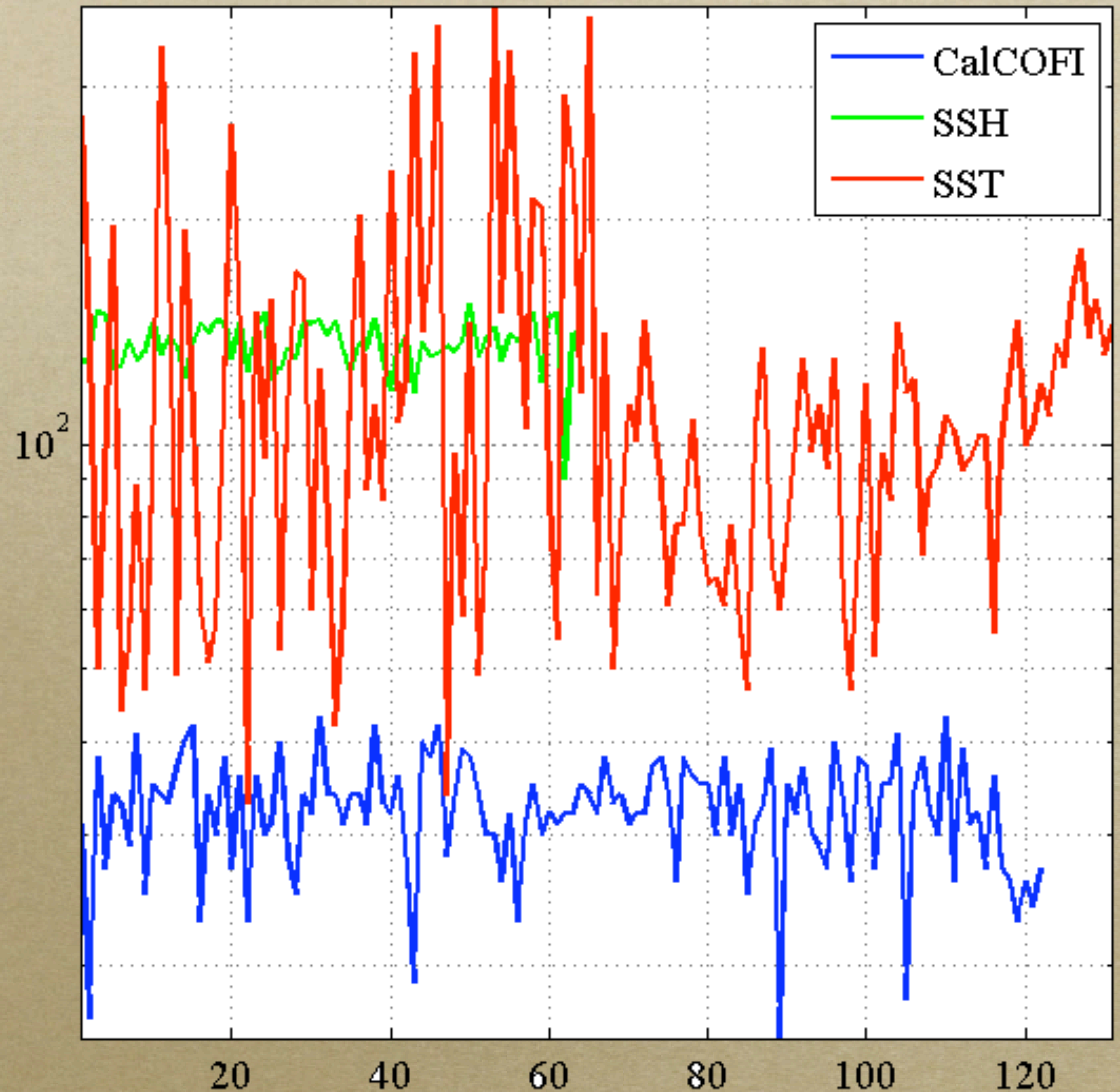
Revised




CalCOFI
130°
125°
120°
115°
40°
35°
30°
0.46
0.43
0.40
0.37
0.34
0.31
0.28
0.25
0.22
0.19
0.16
0.13
0.10
0.07
0.04
0.01
−0.02
−0.05
−0.08
−0.11
−0.14
−0.17
−0.20
−0.23
−0.26
−0.29
−0.32
−0.35
−0.38
−0.41
−0.44
−0.47
−0.50
Min=−4.9937E−01 Max= 4.6480E−01


SSH
130°
125°
120°
115°
40°
35°
30°
0.111
0.104
0.098
0.092
0.086
0.080
0.073
0.067
0.061
0.055
0.048
0.042
0.036
0.030
0.024
0.017
0.011
0.005
−0.001
−0.008
−0.014
−0.020
−0.026
−0.032
−0.039
−0.045
−0.051
−0.057
−0.064
−0.070
−0.076
−0.082
−0.088
Min=−8.8423E−02 Max= 1.1071E−01


SST
130°
125°
120°
115°
40°
35°
30°
6.2
5.8
5.4
4.9
4.5
4.1
3.6
3.2
2.8
2.3
1.9
1.5
1.0
0.6
0.2
−0.2
−0.7
−1.1
−1.5
−2.0
−2.4
−2.8
−3.3
−3.7
−4.1
−4.5
−5.0
−5.4
−5.8
−6.3
−6.7
−7.1
−7.6
x10⁻⁴
Min=−7.5628E−04 Max= 6.2113E−04


CalCOFI
SSH
SST
10^2
20
40
60
80
100
120